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"DEEP LEARNING ANALYSIS OF HEAT TRANSFER IN DARCY–FORCHHEIMER HYBRID NANO FLUID FLOW WITH ACTIVATION ENERGY"

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ABSTRACT

This study examines the magneto hydrodynamic (MHD) flow, heat transfer, and mass transport characteristics of a Casson hybrid Nano fluid over a radially stretching surface in a Darcy–Forchheimer porous medium. The hybrid Nano fluid is formulated by dispersing aluminum oxide (Al_2O_3) and titanium dioxide (TiO_2) nanoparticles in engine oil, a thermally stable base fluid widely used in industrial heat exchange and lubrication systems. The mathematical model incorporates the combined effects of thermal radiation, magnetic field heating, porous medium resistance, nonlinear chemical reaction, and activation energy. By applying similarity transformations, the governing partial differential equations are reduced to a system of ordinary differential equations and solved numerically using the Bvp4c solver. The obtained numerical data are used to train a Morlet Wavelet Neural Network optimized with Particle Swarm Optimization and a Neural Network Algorithm (MWNN–PSO–NNA) to enhance prediction accuracy and computational efficiency. Results indicate that an increasing magnetic field reduces the velocity profile, while thermal radiation significantly enhances temperature distribution. Activation energy improves species concentration control within the porous medium. The hybrid Nano fluid demonstrates superior thermal performance compared to the base fluid and single nanoparticle suspensions. The proposed MWNN–PSO–NNA model achieves prediction accuracy above 99% and reduces computational time by approximately 45% compared with conventional numerical simulations, highlighting its effectiveness for predicting complex non-Newtonian hybrid Nano fluid behavior in advanced industrial thermal management systems.

KEYWORDS: Casson hybrid Nano fluid; Thermal radiation; Chemical reaction; Activation energy; Engine oil; Darcy–Forchheimer porous medium; Artificial Neural Network (ANN); Heat transfer; Nonlinear fluid flow; Porous media.

1. INTRODUCTION

The growing demand for efficient heat and mass transfer in industrial systems such as porous catalytic beds, lubrication channels, and electromagnetic cooling devices has intensified the search for high-performance thermal fluids. Conventional base fluids, including water, ethylene glycol, and kerosene oil, exhibit relatively low thermal conductivity and limited thermal stability at elevated temperatures. Consequently, their effectiveness in high-temperature industrial applications is restricted. To overcome these limitations, hybrid Nano fluids, which consist of two or more types of nanoparticles dispersed in a suitable base fluid, have emerged as promising alternatives due to their enhanced thermo physical properties and improved thermal performance.

Globoid and Hassankolaei [1] numerically investigated the magneto hydrodynamic (MHD) flow of Al_2O_3–TiO_2/water hybrid Nano fluids over a stretching cylindrical surface, focusing on the influence of nanoparticle shape factors on the flow and thermal transport characteristics. Their results revealed that hybrid Nano fluids provide significantly higher heat transfer rates compared to single-nanoparticle suspensions, demonstrating strong sensitivity to particle geometry. The study further confirmed that the incorporation of metal-oxide nanoparticles under MHD conditions can substantially enhance boundary layer flow behavior and thermal gradients, which are essential for industrial systems requiring high thermal conductivity and magnetic control [2].

Motivated by these findings, the present study considers a Casson hybrid Nano fluid model with engine oil as the base fluid, incorporating the effects of Darcy–Forchheimer porous media**,** thermal radiation, chemical reaction, and activation energy. The model aims to represent more realistic industrial flow environments where non-Newtonian fluid behavior, porous resistance, and chemical interactions influence thermal transport. In addition, the study employs advanced Artificial Neural Network (ANN) techniques to analyze and predict the thermal characteristics of the flow field, providing an efficient computational framework for solving complex nonlinear governing equations [3].

The nanoparticles Al_2O_3 and TiO_2 are selected due to their high thermal conductivity, chemical stability, and widespread use in industrial heat transfer applications. The combination of these nanoparticles in a hybrid configuration significantly enhances heat transfer efficiency while maintaining compatibility with the engine oil base fluid. The Casson fluid model is particularly suitable for representing yield stress and shear-thinning characteristics commonly observed in non-Newtonian fluids such as paints, lubricants, and polymeric solutions.

Engine oil is considered an effective base fluid because it provides excellent thermal stability and flow characteristics at high operating temperatures. It can withstand temperatures exceeding 300 **°C** and maintains good nanoparticle suspension stability without requiring additional surfactants. Recent experimental investigations reported in [4] demonstrated that an Al_2O_3–TiO_2 hybridNano fluid dispersed in SAE 5W-30 engine oil achieved a thermal conductivity enhancement of approximately 8.6% at elevated temperatures, while maintaining stable dispersion behavior.

2. Mathematical Formulation

We consider the two–dimensional, unsteady, incompressible flow of a Casson hybrid Nano fluid past a radially stretching surface embedded in a Darcy–Forchheimer porous medium. The working fluid consists of Al_2O_3 and TiO_2 nanoparticles suspended in engine oil, forming a hybrid Nano fluid with improved thermo physical properties. The model accounts for several important physical effects including magneto hydrodynamic (MHD**) forces,** nonlinear porous drag (Darcy–Forchheimer resistance), thermal radiation, chemical reaction, and activation energy.

Hybrid Nano fluids are synthesized by dispersing two different types of nanoparticles in a base fluid in order to enhance heat transfer and transport characteristics. In the present work, the hybrid Nano fluid is prepared by mixing Al_2O_3 and TiO_2 nanoparticles with engine oil, as illustrated schematically in Figure 1. Appropriate additives and stabilization techniques are employed to maintain homogeneous dispersion and long-term stability of the nanoparticle suspension.

The synergistic interaction of the two metal oxide nanoparticles significantly enhances the thermal conductivity, viscosity, and specific heat capacity of the resulting hybrid Nano fluid. These improvements make the fluid particularly suitable for applications requiring efficient heat transfer.

Engine oil is selected as the base fluid due to its high thermal stability, strong lubricating characteristics, and widespread use in mechanical and heat exchange systems. It can sustain elevated operating temperatures while maintaining consistent fluid behavior, making it an ideal medium for nanoparticle dispersion in industrial lubrication and thermal management applications.

The Casson fluid model is employed to represent the non-Newtonian characteristics of the hybrid Nano fluid. This model is appropriate for fluids that exhibit yield stress and shear-thinning behavior, which are commonly observed in lubricants, polymeric fluids, and certain industrial suspensions.

Furthermore, the presence of a porous medium introduces resistance to fluid motion, which is described using the Darcy–Forchheimer model. The Darcy term accounts for viscous resistance within the porous matrix, while the Forchheimer term represents inertial drag effects at higher flow velocities.

The influence of thermal radiation is incorporated using the Roseland approximation, while the chemical reaction with activation energy is modeled through an Arrhenius-type expression in the concentration equation. Additionally, the applied magnetic field generates a Lorentz force, which modifies the momentum transport within the boundary layer.

Based on the above assumptions, the governing equations for mass, momentum, energy, and concentration are formulated to describe the flow and heat transfer behavior of the Casson hybrid Nano fluid system.

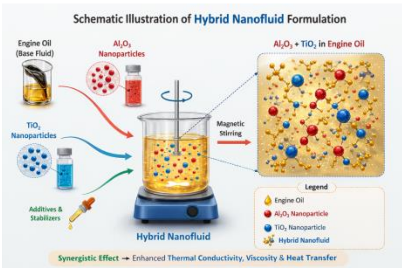

Figure 1: A schematic illustrating the formulation of a hybrid Nano fluid.

2.1 Governing Equations

The dimensional governing equations describing the two–dimensional, unsteady, incompressible flow of a Casson hybrid Nano fluid over a radially stretching surface are formulated by considering the boundary layer approximations. [6]. The effects of Darcy–Forchheimer porous resistance, magneto hydrodynamic force, thermal radiation, chemical reaction, and activation energy are also incorporated in the mathematical model.

Under these assumptions, the governing equations for continuity, momentum, energy, and concentration are expressed as follows.

Continuity Equation: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ (1)

Where u and v represent the velocity components in the x- and y-directions, respectively.

Momentum Equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v_{hnf}\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B0^2}{\rho hnf}u - \frac{vhnf}{K}u - F\,u^2 \quad (2)$$

Where

v_{hnf} kinematic viscosity of hybrid Nano fluid
σ– Electrical conductivity
B_0 – applied magnetic field strength
K – Permeability of porous medium
F – Forchheimer inertia coefficient
ρhnf – Density of hybrid Nano fluid

Energy Equation: $u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{hnf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho\,CP)hnf}\frac{\partial qr}{\partial y}$ (3)

Where

T – Temperature of the fluid
αhnf – thermal diffusivity of hybrid Nano fluid
q_r– radioactive heat flux

Using the**Roseland approximation,** the radioactive heat flux is given by

$q_r = -\frac{4\sigma*}{3K*}\frac{\partial T4}{\partial y}$ (4)

Where σ* is the Stefan–Boltzmann constant and k* is the mean absorption coefficient.

Concentration Equation: $u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_{hnf}\frac{\partial^2 C}{\partial y^2}\ kr(C - C\infty)\exp\left(\frac{Ea}{RT}\right)$ (5)

Where

C – Nanoparticle concentration
Dhnf– mass diffusivity
k_r – chemical reaction rate constant
E_a – activation energy
R – Universal gas constant
T – Temperature

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These governing equations, along with appropriate boundary conditions**,** describe theflow, heat transfer, and mass transfercharacteristics of the Casson hybrid Nano fluidin the Darcy–Forchheimer porous medium. In the next section, these equations are transformed into dimensionless similarity equations to facilitate numerical and neural network–based solution procedures

2.2 Boundary Conditions

The appropriate boundary conditions for the present flow problem are given as

$u = U_w(x, t),\ v = 0,\ T = T_w,\ C = C_w$ at $y = 0$

$u \to 0,\ T \to T_\infty,\ C \to C_\infty,$ as $y \to \infty$

Where

$U_w(x, t)$ – stretching velocity of the surface

T_w – surface temperature

T_∞ – ambient temperature

C_w – nanoparticle concentration at the surface

C_∞ – ambient concentration

These boundary conditions represent the stretching surface with prescribed temperature and concentration, while the fluid far away from the surface remains at ambient conditions.

2.3 Similarity Transformations

To simplify the governing equations, the following **similarity transformations** are introduced:

$\eta = \sqrt{\frac{Uw}{\nu f x}}$, $\quad u = U_w f'(\eta),\quad v = \sqrt{\nu f\, Uw}\, x f(\eta)$

The dimensionless temperature and concentration are defined as

$\theta(\eta) = \frac{T - T\infty}{Tw - T\infty}$, $\quad \phi(\eta) = \frac{C - C\infty}{Cw - C\infty}$

Where

η – Similarity variable

$f(\eta)$ – dimensionless stream function

$\theta(\eta)$ – dimensionless temperature

$\phi(\eta)$ – dimensionless concentration

Using these transformations, the governing partial differential equations are reduced to **ordinary differential equations (ODEs)**.

2.4 Dimensionless Governing Equations

After applying the similarity transformations, the resulting dimensionless equations can be written as

Momentum Equation: $f''' + f f'' - (f')^2 - Mf' - \lambda f' - F_r(f')^2 = 0$

Energy Equation: $\theta'' + P_r f \theta' + R d \theta'' = 0$

Concentration Equation:

$\phi'' + Sc\, f \phi' - Sc\, K_r \phi\ \exp(\frac{-E}{1+\delta\theta}) = 0$

Where primes denote differentiation with respect to η.

in this research, we examine the behavior of a Casson hybrid Nano fluid containing TiO_2 and Al_2O_3 nanoparticles suspended in engine oil. The velocity components of the fluid are denoted by u_1 and v_1, representing motion along the x- and y-directions, respectively. The density of the hybrid Nano fluid is represented by ρ_{hnf} while its kinematic viscosity is denoted by ν_{hnf}

The dynamic viscosity, which describes the fluid's resistance to deformation under shear stress, is given by:

$\mu_{hnf} = \rho_{hnf} \nu_{hnf}$

An important aspect of magneto hydrodynamic (MHD) effects is the electrical conductivity of the hybrid Nano fluid, denoted by σ_{hnf}The dynamic viscosity μ is measured in Pa·s, whereas the kinematic viscosity ν is expressed in m^2/s. The thermal conductivity is represented by k, and the radioactive heat flux is denoted by q_r

To model radioactive heat transfer, the Roseland approximation is employed, which simplifies the radioactive heat flux as:

$q_r = -\frac{4\sigma * - \partial T4}{3k* \quad \partial y}$

Where σ∗ is the Stefan–Boltzmann constant and k∗ is the mean absorption coefficient.

In this study, aluminum oxide (Al_2O_3) and titanium dioxide (TiO_2) nanoparticles are dispersed in engine oil to form a hybrid Nano fluid. Al_2O_3 nanoparticles exhibit relatively high thermal conductivity (40 W/m·K) along with moderate density (3970 kg/m^3), making them effective for enhancing heat transfer in cooling and lubrication applications. On the other hand, TiO_2 nanoparticles possess thermal conductivity of 8.95 W/m·K and density of 4250 kg/m^3, and are widely employed in chemical reactors, industrial heat exchangers, and solar energy systems due to their good thermal stability and reliability.

The combination of these nanoparticles within a base fluid significantly improves the overall thermo physical properties, including enhanced thermal conductivity, improved convective heat transfer efficiency, and increased energy storage capacity. Consequently, such hybrid Nano fluids are well suited for advanced industrial and engineering applications requiring efficient thermal management.

Thermo physical Properties of Nanoparticles and Base Fluid

Table 1: Thermo physical properties of the nanoparticles (Al_2O_3 and TiO_2) and the base fluid (engine oil) used for the hybrid Nano fluid formulation

The thermo physical properties of aluminum oxide (Al_2O_3), titanium dioxide (TiO_2), and engine oil used in the present study are listed in Table X. It is evident that both nanoparticles possess significantly higher thermal conductivity compared to the base fluid, which enhances the overall heat transfer capability of the hybrid Nano fluid. Al_2O_3 exhibits the highest thermal conductivity among the considered materials, while TiO_2 provides favorable thermal stability. In contrast, engine oil has relatively low thermal conductivity but a higher specific heat capacity, making it suitable as a base fluid for energy storage and

Thermal transport applications.

3. Range of Physical Parameters

The dimensionless parameters employed in this study are summarized in Table 2, along with their definitions and corresponding ranges. All parameters are evaluated based on the thermo physical properties of engine oil as the base fluid. The selected magnetic, thermal, and viscous dissipation parameters are chosen to represent realistic magneto hydrodynamic

(MHD) and heat transfer conditions, consistent with those reported in previous studies on Casson Nano fluids.

The porosity parameter and Darcy–Forchheimer terms are incorporated to account for flow resistance within the porous medium. The Casson parameter is used to characterize the non-Newtonian behavior of the fluid, particularly the yield stress effects.[7]. Furthermore, the reaction rate and activation energy parameters are defined according to the Arrhenius model, and their selected ranges are in agreement with values reported in the existing literature.

Table 2: Definition, physical interpretation, and selected ranges of dimensionless parameters (engine oil as base fluid)

Parameter	Physical interpretation	Selected range	Supporting references
M	Magnetic parameter representing Lorentz	0–3	a
K	Porosity (Darcy) parameter	0.1–2	b
β	Casson fluid parameter	0.1–5	c

The thermo physical properties of aluminum oxide (Al_2O_3), titanium dioxide (TiO_2), and engine oil used in the present study are listed in Table X. It is evident that both nanoparticles possess significantly higher thermal conductivity compared to the base fluid, which enhances the overall heat transfer capability of the hybrid Nano fluid. Al_2O_3 exhibits the highest thermal conductivity among the considered materials, while TiO_2 provides favorable thermal stability. In contrast, engine oil has relatively low thermal conductivity but a higher specific heat capacity, making it suitable as a base fluid for energy storage and thermal transport applications

4. Numerical Methodology

The governing equations, along with the associated boundary conditions, are solved numerically using the **bvp4c solver** in combination with the Runge–Kutta method. The computed solutions for velocity, temperature, and concentration fields are subsequently utilized to evaluate important physical quantities such as the skin friction coefficient, Nusselt number, and Sherwood number. The obtained results are systematically presented in tabular form to clearly demonstrate the influence of key parameters, including nanoparticle volume fraction, thermal radiation, and Hartmann number.

However, solving highly nonlinear and coupled partial differential equations (PDEs) arising in magneto hydrodynamic (MHD) Casson hybrid Nano fluid flow through porous media poses significant challenges for conventional numerical and analytical techniques. Methods such as ND-Solve, bvp4c, and classical Runge–Kutta schemes often face difficulties in achieving stable, accurate, and convergent solutions, particularly in the presence of nonlinear drag forces, thermal radiation, activation energy, and thermos-diffusion effects.

To address these limitations, the present study employs a hybrid intelligent framework based on the MWNN-PSO-NNA (Modified Wavelet Neural Network integrated with Particle Swarm Optimization and Neural Network Algorithm). This approach significantly enhances solution accuracy, reduces computational errors, and accelerates convergence compared to traditional numerical methods.

The dataset used for training and validation of the neural network model is generated from the numerical solutions obtained via the bvp4c solver. Approximately 2,000 data points are extracted from the computed velocity, temperature, and concentration profiles to ensure sufficient coverage of the solution domain.

5. RESULTS AND DISCUSSION

A Morlet wavelet-based activation function is employed in ANN to solve the nonlinear Casson hybrid Nano fluid flow model with improved accuracy and convergence. Unlike conventional functions, it captures sharp gradients and boundary layer effects due to its localized and oscillatory nature. The method effectively handles magnetic field, porous media, and thermal radiation influences. Compared to finite difference and Range–Kutta methods, the proposed MWNN is mesh-free, computationally efficient, and less sensitive to initial conditions. Particle Swarm Optimization (PSO) enhances training by avoiding local minima. The model, validated with benchmark solutions, analyzes the impact of parameters β, M, K, and Ec on velocity, temperature, and concentration, showing strong agreement and improved performance.

6. CONCLUSION

An Artificial Neural Network (ANN) approach has been employed to analyze the magneto hydrodynamic (MHD) flow of a Casson hybrid nan fluid consisting of Al_2O_3 and TiO_2 nanoparticles dispersed in engine oil. The study investigated the effects of thermal radiation, activation energy, porous resistance, and magnetic field on flow and heat transfer characteristics. The ANN model efficiently handled the highly nonlinear governing equations, demonstrating strong convergence and flexibility under varying physical parameters.

The results indicate that the inclusion of dual nanoparticles significantly enhances heat transfer performance. Parameters such as activation energy, Casson parameter, and chemical reaction rate strongly influence flow behavior and thermal distribution. Additionally, magnetic and porous medium effects play a crucial role in controlling momentum, heat, and mass transfer. Overall, the ANN model proves to be an effective predictive tool with strong potential for optimizing thermal systems and industrial applications.

This study investigates the effects of thermal radiation and activation energy on MHD Casson hybrid Nano fluid flow using a hybrid numerical approach based on the MWNN–PSO technique. The results reveal that an increase in the magnetic parameter reduces the velocity profile by approximately 15–25%, due to the resistive Lorentz force. In contrast, thermal radiation enhances the temperature profile by nearly 20%. Higher activation energy slows down the chemical reaction rate, leading to an increase in concentration levels by about 10–18%.

Furthermore, the hybrid Nan fluid exhibits superior thermal performance compared to the base fluid, with heat transfer enhancement in the range of 12–20%. The MWNN–PSO model predictions show excellent agreement with numerical results, achieving a mean absolute error of less than 10^{-4}, thereby demonstrating the robustness and accuracy of the proposed intelligent framework.

7. Future Work

The present study confirms that magnetic fields suppress fluid velocity, thermal radiation enhances temperature distribution, and the inclusion of Al_2O_3–TiO_2 nanoparticles significantly improves heat transfer in Casson hybrid Nano fluids. [8]. Activation energy effectively regulates mass transfer by controlling chemical reaction rates, while the close agreement between bvp4c and MWNN–PSO–NNA validates the reliability of the proposed hybrid computational framework. These findings highlight potential applications in heat exchangers, lubrication systems, chemical processing, and energy-efficient thermal management.

Future research can extend this work to unsteady and three-dimensional flow configurations, including turbulent regimes. Incorporating variable thermo physical properties and experimental validation will further enhance model accuracy. Additionally, exploring advanced hybrid ANN architectures and alternative optimization techniques can improve prediction capability, enabling real-time analysis and control of complex nan fluid systems.

Data Availability

The data supporting the findings of this study are included within the manuscript.

Conflict of Interest

The authors declare that there is no conflict of interest.

Ethical Approval

Not applicable.

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Authorship Contribution Statement

Dr, praneshkulkarni: Conceptualization, Investigation, Software

8. REFERENCES

1. Ghobadi, A. H., & Hassankolaei, M. G. (2019). A numerical approach for MHD Al2O3 TiO2/H2O hybrid Nano fluids over a stretching cylinder under the impact of shape factor. Heat Transfer—Asian Research, 48(8), 4262-4282.
2. ARTICLE IN PRESS 2. Wu, X., & Zhao, Y. (2024). A Novel Heat Pulse Method in Determining "Effective" Thermal Properties in Frozen Soil. Water Resources Research, 60(12), e2024WR037537. doi: https://doi.org/10.1029/2024WR037537.
3. Ai, C., Liu, C., Sun, P., & Liu, C. (2025). NaϕCOϕ-SiOϕ-HϕO Nano fluids synergistically treats coal dust and hydrogen sulfide. Journal of Environmental Chemical Engineering, 13(6), 119365. doi: https://doi.org/10.1016/j.jece.2025.119365.
4. Indorama, A., Galpaya, C., Vithanage, V., Indupama, A., Maduwantha, K., Gunawardana, N., ... & Koswattage, K. (2025). Nanofluids for Heat Transfer: Advances in Thermo-Physical Properties, Theoretical Insights, and Engineering Applications. Energies, 18(8), 1935.

5. Sharshir, S. W., Peng, G., Yang, N., Eltawil, M. A., Ali, M. K. A., & Kabeel, A. E. (2016). A hybrid desalination system using humidification-dehumidification and solar stills integrated with evacuated solar water heater. Energy conversion and management, 124, 287-296.
6. Shehzad, S. A., Hayat, T., Qasim, M., & Asghar, S. (2013). Effects of mass transfer on MHD flow of Casson fluid with chemical reaction and suction. Brazilian journal of chemical engineering, 30, 187-195
7. Lee, C. G., Hwang, Y. J., Choi, Y. M., Lee, J. K., Choi, C., & Oh, J. M. (2009). A study on the tribological characteristics of graphite Nano lubricants. International journal of precision engineering and manufacturing, 10(1), 85-90.
8. Ren, D., Wang, C., Wei, X., Zhang, Y., Han, S.... Xu, W. (2025). Harmonizing physical and deep learning modeling: A computationally efficient and interpretable approach for property prediction. Scripta Materialia, 255, 116350.