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DEVELOPMENT OF A KINEMATIC IDENTIFICATION FRAMEWORK FOR THE DIRECTIONAL MOTION OF A FIXED-WING AIRCRAFT BASED ON AN ARMAX MODEL

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ABSTRACT

This paper presents the development of a kinematic identification framework for the directional motion of a fixed-wing aircraft based on the ARMAX (Auto-Regressive Moving Average with eXogenous inputs) model. The proposed approach aims to accurately identify the dynamic relationship between control inputs and the aircraft's directional motion variables by utilizing measured flight data. First, the kinematic equations governing the directional motion of the fixed-wing aircraft are established. Subsequently, an ARMAX model structure is constructed to represent the system dynamics, in which appropriate model orders are selected to balance accuracy and computational efficiency. The model parameters are identified using input–output data obtained from simulation or experimental flight tests. The effectiveness of the proposed identification framework is evaluated through numerical simulations, where the identified model demonstrates good agreement with the reference data in both transient and steady-state responses. The results confirm that the ARMAX-based kinematic identification framework is capable of capturing the essential characteristics of the aircraft's directional motion and can serve as a reliable basis for further control system design and performance analysis.

INTRODUCTION

Parameter Identification of the Mathematical Model of a Controlled Plant Based on Transient Response

Basis for Developing the Identification Algorithm

Let (h(t)) denote the transient function of the plant, that is, the response of the plant when it is excited by a unit step signal (1(t)) at the input.

$$1(t) = \begin{cases} = 1 \; when \; t > 0 \\ = 0 \; when \; t < 0 \end{cases} \qquad (1)$$

On the other hand, modeling a plant essentially involves describing the mapping between the input signal u(t) and the output signal y(t): TM: $u(t) \rightarrow y(t)$. For a linear system, this mapping can be described by an integral:

$$y(t) = \frac{d}{dt} \int h(t - \tau)\, u(\tau) d\tau \qquad (2)$$

That is, through h(t) , the output y(t) can always be determined from the input signal u(t). Therefore, the transient function h(t) provides a complete description of the plant and can thus be regarded as a nonparametric model of the plant. Similarly, if g(t) denotes the weighting function, that is, the response of the plant when it is excited by an impulse input $\delta(t)$ at the input:

$$\delta(t) = \frac{dh(t)}{dt}$$

Then the mapping TM: $u(t) \rightarrow y(t)$, which describes the input–output relationship, can be expressed as:

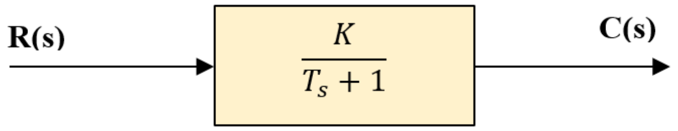
$$y(t) = u(t)h(0) + \frac{d}{dt} \int g(t - \tau)\, u(\tau) d\tau \qquad (3)$$

In other words, similar to h(t), through g(t) the output y(t) can always be obtained from the input u(t); therefore, g(t) can also be considered a nonparametric model of the plant.

Thus, identifying a nonparametric model is equivalent to identifying either the transient function h(t) or the weighting function g(t). Accordingly, when h(t) is determined by exciting the plant with a unit step signal 1(t), the approach is referred to as an active identification method. In contrast, when the nonparametric model is identified by determining g(t) or its Fourier transform G(j*w*) through spectral analysis of the input and output signals, the approach is referred to as a passive identification method.

Determination of the parameters of a first-order inertial model from the transient response

Model:



Transfer function of a first-order inertial element is showed that:

$$G(s) = \frac{K}{T_s + 1}$$

To identify a first-order inertial element, it is necessary to determine the parameters K and T. In order to determine these two parameters, the transient characteristics of the first-order inertial element are utilized.

The transient response of a first-order inertial element is expressed as follows:

$$C(s) = R(s).G(s) = \frac{K}{s.(T_s + 1)}$$

$$c(t) = K(1 - e^{\frac{-t}{T}}) \quad (4)$$

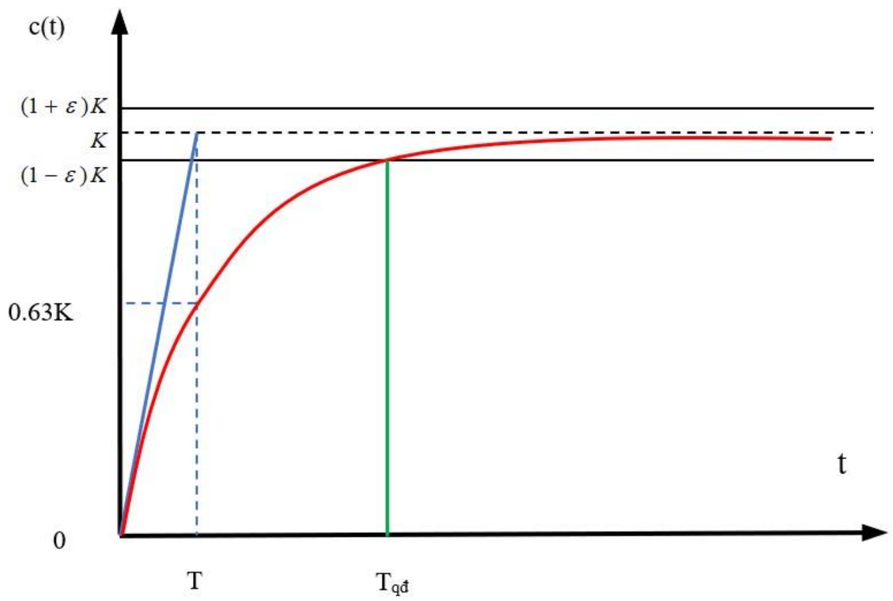


Figure 1. Transient response of a first-order inertial element.

- K is determined as follows

$$K = \lim_{t\to\infty} c(t) = \lim_{s\to 0} G(s) \quad (5)$$

- Determination of (T): From the expression of the transient time:

$$t_{qd} = T.\ln\frac{1}{\varepsilon} \quad (6)$$

+ ε = 0.02 according to the criterion 2%;

+ ε = 0.05 according to the criterion 5%;

+ T is the time instant at which the transient response reaches 63% of the steady-state value

Determination of the parameters of a damped second-order oscillatory model from the transient response

Consider a linear plant to be identified; when it is actively excited at the input by a unit step signal u(t) = 1(t) , the experimentally obtained transient response h(t) at the output has a form similar to that shown in figure 2.

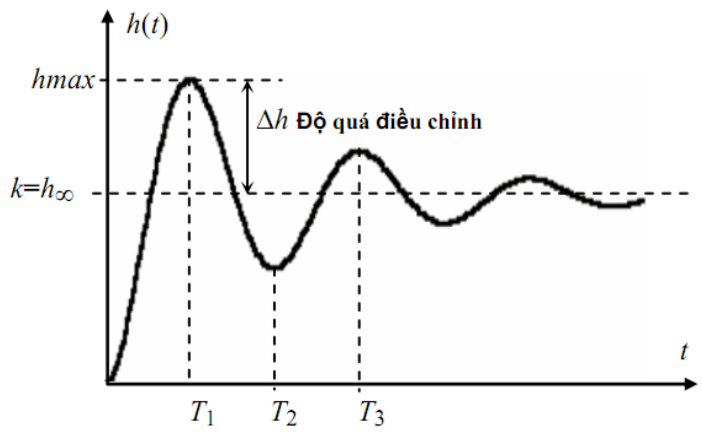


Figure 2. Transient response of a second-order oscillatory plant

Consider the plant described by the model:

$$W(s) = \frac{kq^2}{s^2+2qDs+q^2}\,(0 < D < 1) \quad (7)$$

From this model, it is directly obtained that:

$$h_\infty = \lim_{t\to\infty} h(t) = \lim_{s\to\infty} W(s) = k \quad (8)$$

Therefore, the remaining task is only to determine D and q. Let s_1 and s_2 denote the two poles of model (7); we have:

$$s_{1,2} = -Dq \pm jq\sqrt{1-D^2}$$

Hence $h(t) = k\left[1 - \frac{e^{-Dqt}}{\sqrt{1-D^2}} sin(\,q\sqrt{1-D^2}t + arccosD)\right]$

Và $\frac{dh(t)}{dt} = k\frac{qe^{-Dqt}}{\sqrt{1-D^2}} sin\left(q\sqrt{1-D^2}t\right)$

By solving the equation $\frac{dh(t)}{dt} = 0$ to determine the extrema (including at t = 0), the following is obtained:

$$T_i = \frac{i\pi}{q\sqrt{1-D^2}},\, i = 0,1,\ldots. \quad (9)$$

thus

$$h\,max = h(T_1) = k\left[1 - \frac{sin(\pi + arccos\,D)}{\sqrt{1-D^2}} exp\left(\frac{-\pi D}{\sqrt{1-D^2}}\right)\right] = k\left[1 + exp\left(\frac{-\pi D}{\sqrt{1-D^2}}\right)\right] \quad (10)$$

By subtracting (10) from (8), the overshoot is obtained:

$$\Delta h = h\,max - h_\infty = k\,exp\left(\frac{-\pi D}{\sqrt{1-D^2}}\right) \tag{11}$$

If equation (11) is divided by equation (8) term by term, the following is obtained:

$$\left|\frac{\Delta h}{k}\right| = exp\left(\frac{-\pi D}{\sqrt{1-D^2}}\right) \Leftrightarrow D = \frac{1}{\sqrt{1+\frac{\pi^2}{ln^2\left|\frac{\Delta h}{k}\right|}}} \tag{12}$$

And equation (12) is precisely the formula that allows the parameter D to be determined from the experimental curve h(t). the remaining parameter q is then determined from D with the assistance of T_1 according to (9) as follows:

$$T_1 = \frac{\pi}{q\sqrt{1-D^2}} \Leftrightarrow q = \frac{\pi}{T_1\sqrt{1-D^2}} \tag{13}$$

The three expressions (8), (12), and (13) provide the basis for determining the parameters k, D, and q of model (7) from the experimental curve h(t). If for some reason, the transient function h(t) is not available and only the response y(t) is obtained when the plant is excited by $u(t) = u_0(t)$, then, since the plant is linear, the above expressions undergo a slight modification and become:

$$y_\infty = \lim_{t\to\infty} y(t) = ku_0 \tag{14}$$

$$D = \frac{1}{\sqrt{1+\frac{\pi^2}{ln^2\left|\frac{\Delta h}{k}\right|}}} \tag{15}$$

$$q = \frac{\pi}{T_1\sqrt{1-D^2}} \tag{16}$$

DIRECTIONAL MOTION MODEL OF A FIXED-WING AIRCRAFT

Mathematical model of aircraft motion

When an aircraft is in motion, various forces act on the aircraft, as illustrated in figure 3

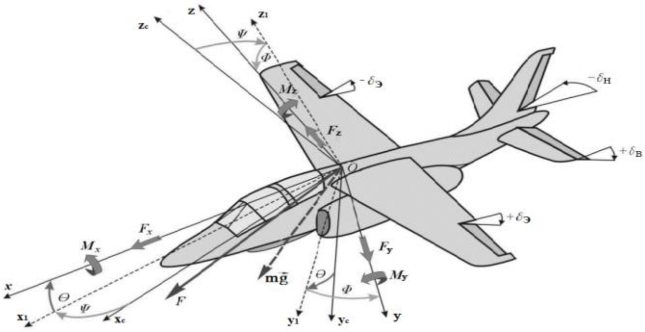


Figure 3. Forces acting on the aircraft during motion

By applying the law of momentum and the law of angular momentum and projecting them onto the body-fixed coordinate system, six Euler equations of rigid-body motion are obtained, which describe the relationships between the kinematic variables and the forces and moments acting on the rigid body.

The three force equations governing the translational motion of the center of mass are:

$$m(\dot{W}_x + \omega_y W_z - \omega_z W_y) = \sum F_x$$
$$m(\dot{W}_y + \omega_z W_x - \omega_x W_z) = \sum F_y \qquad (17)$$
$$m(\dot{W}_z + \omega_x W_y - \omega_y W_x) = \sum F_z$$

The three moment equations representing the rotational motion about the center of mass are:

$$J_x\dot{\omega}_x + (J_z - J_y)\omega_y\omega_z = \sum M_x$$
$$J_y\dot{\omega}_y + (J_x - J_z)\omega_x\omega_z = \sum M_y \quad (18)$$
$$J_z\dot{\omega}_z + (J_y - J_x)\omega_x\omega_y = \sum M_z$$

Where:

- W_x, W_y, W_z - The components of the cruising velocity vector in the body-fixed coordinate system;

- $\omega_x, \omega_y, \omega_z$ - The components of the angular velocity vector in the body-fixed coordinate system;

- m - The mass of aircraft;

- $\sum F_x, \sum F_y, \sum F_z$ and $\sum M_x, \sum M_y, \sum M_z$ - The summation of force components and moments acting on the aircraft in the body-fixed coordinate system;

- J_x, J_y, J_z - The aircraft moments of inertia about the corresponding axes of the body-fixed coordinate system.

By analyzing the system of equations (17) and (18), it is possible to:

Determine the relationships between the forces acting on the aircraft, the flight trajectory, and the motion parameters (such as velocity and the aircraft's angular position in space);

Identify the characteristics of aircraft stability and controllability under different flight regimes.

The system of equations (17) and (18) can be decomposed into longitudinal motion and lateral motion. This decomposition is based on the assumption that the aircraft has a symmetric structure, such that variations in longitudinal motion parameters have only a minor influence on lateral motion parameters, and vice versa.

Longitudinal motion consists of the translational motion of the aircraft along the axes Ox_1 and Oy_1, together with the rotational motion about the Oz_1 axis. In other words, longitudinal motion describes the maneuvering of the aircraft within the vertical plane aligned with the aircraft's longitudinal axis, characterized by the position coordinate x, altitude H, angle of attack α, pitch angle θ, and flight path angle γ.

Accordingly, the equations governing longitudinal motion are given by:

$$\begin{cases} m(\dot{W}_z + \omega_y W_z - \omega_z W_y) = \sum F_{x1} \\ m(\dot{W}_x + \omega_z W_x - \omega_x W_z) = \sum F_{y1} \\ J_z \dot{\omega}_y + (J_y - J_x)\omega_x \omega_y = \sum M_{z1} \end{cases} \quad (19)$$

Lateral motion refers to the motion of the aircraft along the Oz_1 axis and its rotation about the Ox_1 and Oy_1 axes. In other words, lateral motion describes the maneuvering of the aircraft in the horizontal plane, characterized by the lateral displacement Z, bank angle γ, heading angle ψ, side slip angle β, and turn (trajectory) angle θ_Γ.

Accordingly, the corresponding equations are given by:

$$\begin{cases} m(\dot{W}_z + \omega_x W_y - \omega_y W_x) = \sum F_z \\ J_x \dot{\omega}_x + (J_z - J_y)\omega_y \omega_z = \sum M_x \\ J_y \dot{\omega}_y + (J_x - J_z)\omega_x \omega_z = \sum M_y \end{cases} \quad (20)$$

After decomposing the aircraft motion into longitudinal and lateral motions to facilitate the analysis of stability and control characteristics, the equations of motion are linearized.

Linear mathematical model of the aircraft yaw dynamics

To develop the mathematical model of the yaw channel, it is assumed that the longitudinal and lateral motions of the aircraft are decoupled. Under this assumption, the equations governing the lateral–directional motion are derived.

It is assumed that the aircraft is in steady, wings-level flight, with no sideslip and no external disturbances: $\Delta W = 0; \Delta H = \Delta Y = 0; \Delta P = 0; \gamma_0 = \beta_0 = 0; \Delta X = 0.$

By linearizing the moment equations, the following linear system of equations governing the lateral–directional motion is obtained:

$$\begin{cases} -\dot{\theta}_\Gamma + a_z^{\gamma}\gamma + a_z^{\beta}\beta = a_z^{\beta_b}\beta_b \\ \ddot{\gamma} + a_{m_x}^{\omega_x}\dot{\gamma} + a_{m_x}^{\omega_y}\dot{\psi} + a_{m_x}^{\beta}\beta = a_{m_x}^{\delta_э}(-\delta_э) \\ \ddot{\psi} + a_{m_y}^{\omega_x}\dot{\gamma} + a_{m_y}^{\omega_y}\dot{\psi} + a_{m_y}^{\beta}\beta = a_{my}^{\delta_H}(-\delta_H) \\ \psi = \theta_\Gamma + \beta - \beta_b \end{cases} \quad (21)$$

System (21) represents the complete general linearized equations of the aircraft lateral–directional motion.

From system (21), with $\gamma = 0$, the independent equation of motion in the yaw direction is obtained as follows:

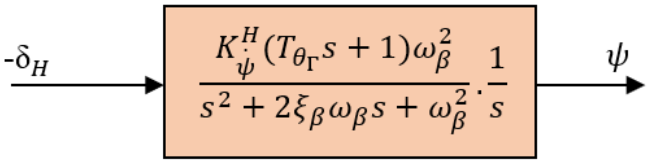
$$\begin{cases} -\dot{\theta}_\Gamma + a_z^{\beta}\beta = 0 \\ \ddot{\psi} + a_{m_y}^{\omega_y}\dot{\psi} + a_{m_y}^{\beta}\beta = a_{m_y}^{\delta_H}(-\delta_H) \\ \psi = \theta_\Gamma + \beta \end{cases} \quad (22)$$

The transfer function from the control input to the yaw angle of the aircraft is obtained as follows:

$$W_{\psi}^{\delta_H}(s) = \frac{\psi(s)}{-\delta_H(s)} = W_{\dot{\psi}}^{\delta_H}(s).\frac{1}{s} = \frac{K_{\dot{\psi}}^{H}(T_{\theta_\Gamma}s+1)\omega_\beta^2}{s^2+2\xi_\beta\omega_\beta s+\omega_\beta^2}.\frac{1}{s} \quad (23)$$

$K_{\dot{\psi}}^{H} = \frac{K_{\beta}^{H}}{T_{\theta_\Gamma}}$; $T_{\theta_\Gamma} = \frac{1}{a_z^{\beta}}$; T_{θ_Γ} is the time constant associated with the turn angle.

Based on the transfer function, the block diagram is derived as follows:



$-\delta_H$, ψ: The rudder deflection angle and the aircraft yaw angle.

Statement of the ARMA model identification problem

Parameter identification of the ARMA (Auto-Regressive Moving Average) model is a parameter estimation approach for discrete-time systems.

Consider the following discrete-time model:

$$G(z) = \frac{Y(z)}{U(z)} = K\frac{1+b_1z^{-1}+...+b_{nb}z^{-nb}}{1+a_1z^{-1}+...+a_{na}z^{-na}} \quad (24)$$

Based on the observation and measurement of input and output signals, the model parameters are estimated so as to minimize the discrepancy between the model and the actual system. Different formulations of the modeling error lead to different identification methods. These methods are generally classified into two main categories: active identification and passive identification.

In particular, when n_b=0, model (24) reduces to:

$$G(z) = \frac{K}{1+a_1 z^{-1}+\ldots+a_{na} z^{-na}} \tag{25}$$

This model is referred to as the Auto-Regressive (AR) model

When na=0, model (24) reduces to:

$$G(z) = K(1 + b_1 z^{-1}+\ldots+b_{nb} z^{-nb}) \tag{26}$$

This model is referred to as the Moving Average (MA) model

By combining the AR and MA models, the Auto-Regressive Moving Average (ARMA) model is obtained.

ARMAX model

The general equation of the ARMAX model is given by:

$$y(k) = \frac{B(q)}{F(q)} u(k) + \frac{C(q)}{D(q)} e(k) \tag{27}$$

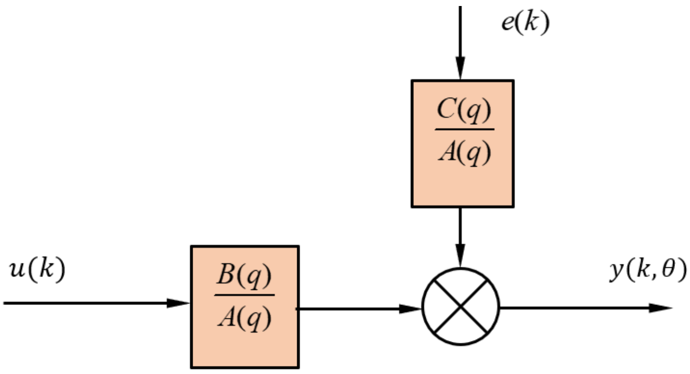


Figure 2. Block diagram of the ARMAX identification model.

$$D(q) = F(q) = A(q)$$

$$\Rightarrow A(q)y(k) = B(q)u(k) + C(q)e(k) \tag{28}$$

Where:

$$A(q) = 1 + a_1 q^{-1} + \cdots + a_{na} q^{-na}$$

$$B(q) = b_1 q^{-nk} + b_2 q^{-nk-1} + \cdots + b_{nb} q^{-nk-nb-1}$$

$$C(q) = 1 + c_1 q^{-1}+\ldots+c_{nc} q^{-nc} \tag{29}$$

The ARMAX model predictor can be expressed in the form of a pseudo-linear regression:

$$\hat{y}(k,\theta) = \varphi^T(k,\theta)\theta \tag{30}$$

$$\theta = [a_1 \ldots a_{na} b_1 \ldots b_{nb} c_1 \ldots c_{nc}]^T \quad (31)$$

$$\varphi(k,\theta) = [-y(k-1) \ldots - y(k-na) u(k-nk) \ldots$$
$$u(k-nk-nb+1) \varepsilon(k-1,\theta) \ldots \varepsilon(k-nc,\theta)]^T \quad (32)$$

$$\varepsilon(k,\theta) = y(k) - \hat{y}(k,\theta) \quad (33)$$

ARMAX model: sys = armax(data, [na nb nc nk])

Where na, nb, nc, nk denotes the model order, q−1is the time-delay operator, and the ARMAX model parameters a_k, b_k, and c_k are defined as given in the corresponding equations.

$$C(q) = 1 + c_1 q^{-1} + \ldots + c_{nc} q^{-nc}$$

In contrast to the ARX model, the ARMAX model incorporates the parameter C(q)

MATERIAL AND METHODS

DEVELOPMENT OF A KINEMATIC IDENTIFIER FOR THE YAW MOTION OF A FIXED-WING AIRCRAFT USING THE ARMAX MODEL

Identification Data Acquisition

Assume that the yaw motion model of the aircraft is available, but its internal parameters are unknown. This model is represented as a single block in Simulink. The block has one input and one output, as illustrated below:

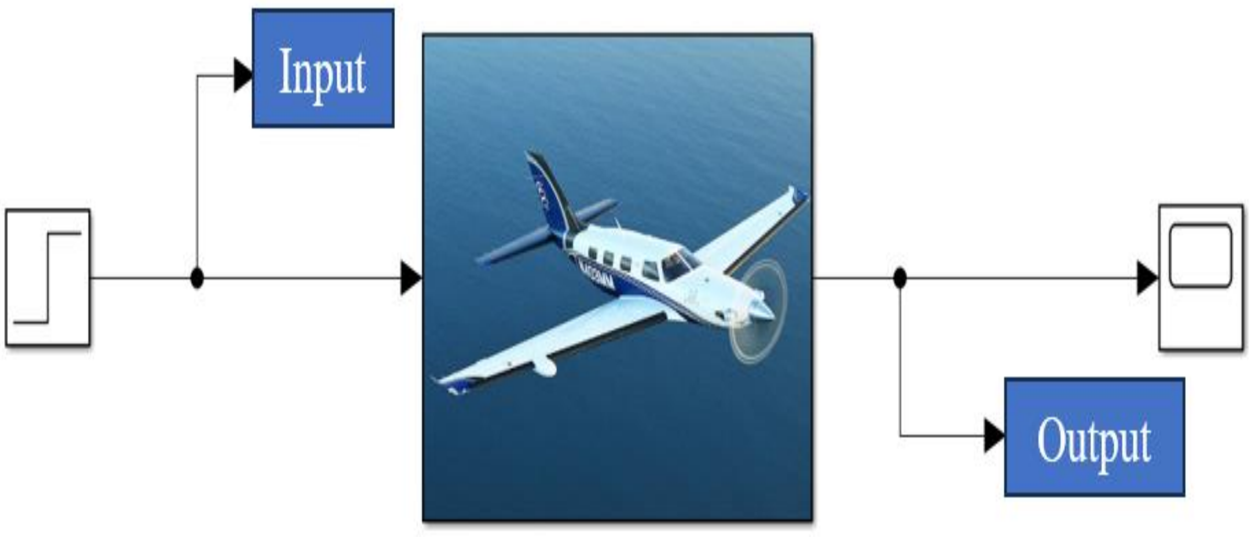


Figure 2.2. Reference model of the yaw motion channel of a fixed-wing aircraft

To acquire identification data, a unit-step input signal 1(t) is applied to the model, and the corresponding output response is recorded. Two *To Workspace* blocks, named **'Input'** and **'Output'**, are used to export the input and output data to the MATLAB workspace.

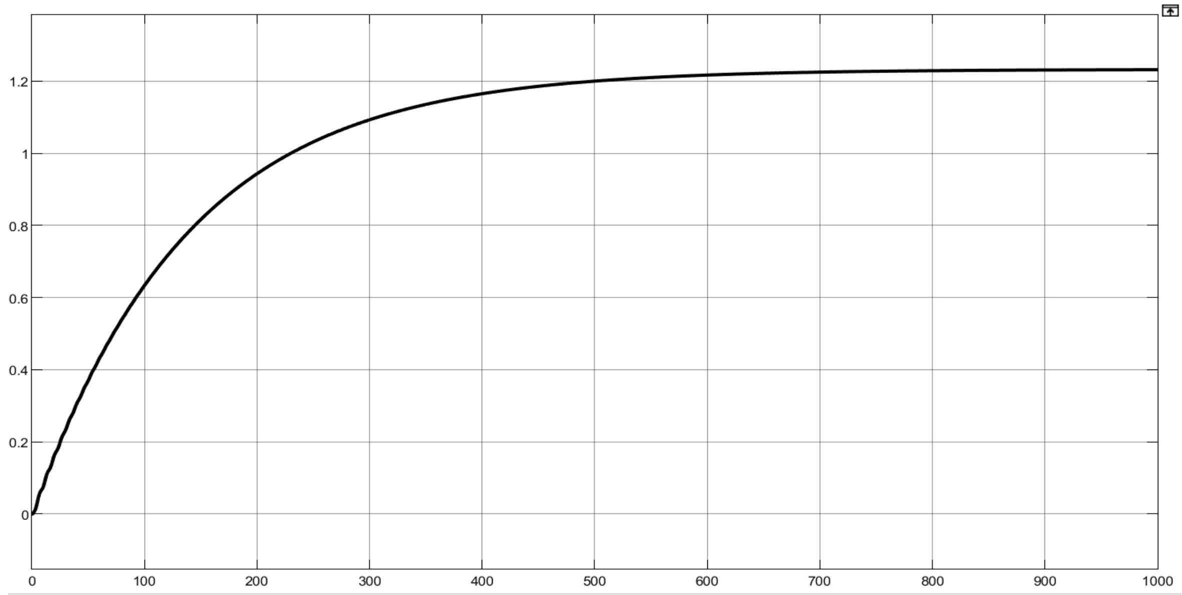


Figure 2.3. Transient response of the model

Model identification

The identification methods and algorithms have been presented above; however, the system cannot be identified directly using the ARMAX model. Instead, a combined identification procedure involving the Auto-Regressive with eXogenous input (ARX) model and the Moving Average (MA) model is employed. This is because the ARMAX model includes the parameter C(q), which is not present in the ARX model.

Assume that $C(q) = 1$ is given; the predicted output $\hat{y}(k,\theta)$ is computed using equation (7) with the available input–output data and the identified A(q) and B(q) polynomials. Subsequently, $\varepsilon(k,\theta)$ is calculated according to equation (33). At this stage, a complete data set is obtained for applying the ARMAX identification algorithm.

Identification program: *nhandangARMAX.m*

```
n= length (Input);
U= Input (1: n,1);
Y= Output (1: n,1);
t= tout (1: n,1);
T=0.01;
Phi = [-Y U]';
theta=inv(phi*phi') * phi*Y;
y_est=phi'*theta;
```

```
error1=Y-y_est;
phi1 = [-Y U error1]';
theta1=inv(phi1*phi1') *phi1*Y;
y_est1=phi1'*theta1;
error2=Y-y_est1;
plot (t, Y,'b', t, y_est1,'r--','LineWidth',2.0);
legend('Thucnghiem','Nhandang');
title ('Ket qua nhan dang');
```

The identification results are as follows:

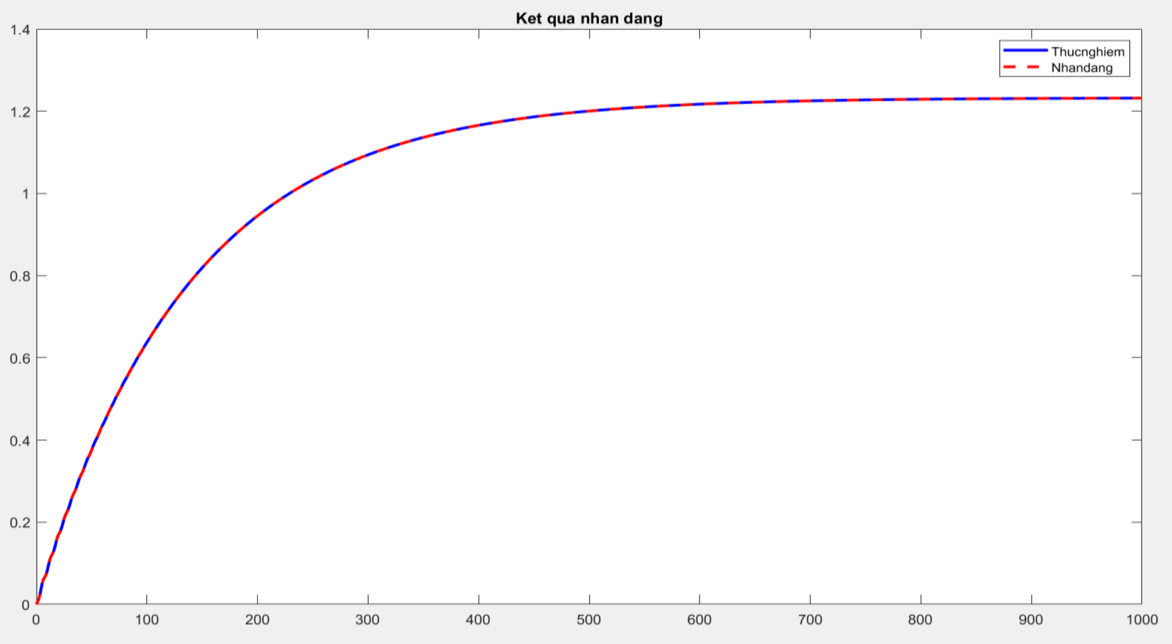


Figure 2.4. Identification results using the ARMAX model.

DISCUSSION AND CONCLUSION

The identified ARMAX model accurately reproduces the transient characteristics of the system, yielding an extremely small mean square error $error2 = 3767e^{-22}$ and an identification fit of approximately 100%.

Furthermore, the parameters of the ARMAX model are successfully estimated.

$$\theta = [-1.00005.0680e^{-11}4.1402e^{3}]$$

Correspondingly: $A(z) = 1 - 0000z^{-1}$

$$B(z) = 5.0680e^{-11}z^{-1}$$

$$C(z) = 1 + 4.1402e^{3}z^{-1}$$

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