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PATTERNS OF INTEGER SOLUTIONS TO NON-HOMOGENEOUS TERNARY QUARTIC EQUATION

*[1]N.Thiruniraiselvi, [2]J.Shanthi, [3]M. A. Gopalan,

[1]Associate Professor, Department of Mathematics, Sri Ramakrishna College of Engineering, Affiliated to Anna University, Chennai, Sri Saradha Nagar, Perambalur-621 113, Tamil Nadu, India.

[2]Assistant Professor of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Tiruchirappalli, Tamil Nadu, India.

[3]Professor of Mathematics, Shrimati Indira Gandhi College, Affiliated to Bharathidasan University, Tiruchirappalli, Tamil Nadu, India.

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***Corresponding Author: N.Thiruniraiselvi**
Associate Professor, Department of Mathematics, Sri Ramakrishna College of Engineering, Affiliated to Anna University, Chennai, Sri Saradha Nagar, Perambalur-621 113, Tamil Nadu, India. DOI: https://doi-doi.org/101555/ijrpa.8247

ABSTRACT

This communication focuses on determining distinct integer solutions to non-homogeneous polynomial equation of degree four with three unknowns given by $x^4 + x^2 + y^2 - y = z^2 + z + 9k^2 - 15k + 6$. Substitution technique and factorization method are employed to obtain the same.

KEYWORDS : Non-homogeneous quartic , Ternary quartic ,Integer solutions, Substitution technique, Factorization method.

INTRODUCTION

The subject of Diophantine equations is one of the areas of number theory that has attracted many mathematicians. It is well-known that Diophantine equations are rich in variety. In particular , homogeneous and non-homogeneous Diophantine equations of degree four with multi-variables have aroused the interest of numerous mathematicians since antiquity. For simplicity and brevity, a few fourth degree Diophantine equations with multi-variables are presented in [1-13] . In this communication , an interesting non-homogeneous polynomial Diophantine equation of degree four with three unknowns given by $x^4 + x^2 + y^2 - y = z^2 + z + 9k^2 - 15k + 6$ is considered for obtaining patterns of integer solutions. Substitution technique and factorization method are employed to obtain the same.

Method of analysis

The non-homogeneous ternary bi-quadratic equation to be solved is

$$x^4 + x^2 + y^2 - y = z^2 + z + 9k^2 - 15k + 6 \qquad (1)$$

The process of obtaining patterns of integer solutions to (1) is illustrated below:

Process 1

Inserting

$$z = x^2 \qquad (2)$$

in (1) ,we get

$$y^2 - y - (9k^2 - 15k + 6) = 0 \qquad (3)$$

Treating (3) as a quadratic equation in y and solving for the same, one has

$$y = \frac{[1 \pm \sqrt{1 + 4(9k^2 - 15k + 6)}]}{2}$$

$$= \frac{[1 \pm (6k - 5)]}{2}$$

$$= (3k - 2) , (3 - 3k)$$

Thus, the integer solutions to (1) are given by

$$x = s, y = 3k - 2, z = s^2$$

$$x = s, y = 3 - 3k, z = s^2$$

Process 2

Taking

$$y = z + 1 \qquad (4)$$

in (1) , one has

$$x^4 + x^2 - (9k^2 - 15k + 6) = 0 \qquad (5)$$

Treating (5) as a quadratic in x^2 and solving for the same, we have

$$x^2 = \frac{[-1 \pm (6k - 5)]}{2}$$

$$= (3k - 3), (2 - 3k)$$

After some algebra , it is seen that the expression $(3k-3)$ is a perfect square when

$$k = 3s^2 + 1 \tag{6}$$

and

$$x = \pm 3s \tag{7}$$

Thus , (1) is satisfied by

$$x = \pm 3s, y = \alpha + 1, z = \alpha, k = 3s^2 + 1$$

Remark

It is to be noted that there is no integer value of k such that the expression $2-3k$ is a perfect square.

Process 3

On completing the squares , (1) is written as

$$X^2 + Y^2 = [Z^2 + (6k-5)^2]*1 \tag{8}$$

where

$$X = 2x^2 + 1 , Y = 2y - 1 , Z = 2z + 1 \tag{9}$$

Assume the integer 1 on the R.H.S. of (8) as

$$1 = \frac{(3+4i)(3-4i)}{25} \tag{10}$$

Substitute (10) in (8). Employing factorization and equating the positive factors, one has

$$X + iY = [Z + (6k-5)i]*\frac{(3+4i)}{5}$$

On comparing the coefficients of corresponding terms , we get

$$\left.\begin{aligned} X &= \frac{1}{5}[3Z - 4(6k-5)] \\ Y &= \frac{1}{5}[4Z + 3(6k-5)] \end{aligned}\right\} \tag{11}$$

Using the value of Z from (9) in (11) ,we have

$$\left.\begin{aligned} X &= \frac{1}{5}[6z - 24k + 23] \\ Y &= \frac{1}{5}[8z + 18k - 11] \end{aligned}\right\} \tag{12}$$

Employing (9) in (12) for X ,Y, we have

$$x^2 = \frac{3}{5}[z - 4k + 3]$$
$$y = \frac{1}{5}[4z + 9k - 3] \quad (13)$$

After some algebra, it is seen that the values of x ,y in (13) are integers when

$$z = 15\alpha^2 + 4k - 3 \quad (14)$$

and

$$x = \pm 3\alpha, y = 12\alpha^2 + 5k - 3 \quad (15)$$

Thus,(14) & (15) satisfy (1).

Interesting observations:

1.

2.

3.

4.

is a nasty number k is a perfect square integer

5.

is a perfect square for values of k given by

$k = 3s^2 - 4s + 2, 3s^2 - 2s + 1$

6.

is a perfect square for values of k given by

k= 1,4,5,12,20,33,36,53,69,92,97,124,…

CONCLUSION

This paper illustrates the process of obtaining varieties of non-zero distinct integer solutions to non-homogeneous ternary bi-quadratic equation given by $x^4 + x^2 + y^2 - y = z^2 + z + 9k^2 - 15k + 6$. The methods presented in this paper may be utilized to obtain integer solutions to higher degree Diophantine equations with three unknowns.

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