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PERFORMANCE ANALYSIS OF PID-LQR CONTROLLED FLEXIBLE LINK MANIPULATOR FOR VARIED MATRIX GAIN OF OPTIMAL CONTROLLER

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ABSTRACT:
The analysis conducted via computer simulation carried out in MATLAB/Simulink environment for the developed PID-LQR control system of Flexible Link Robot Manipulator reviewed that the combination of PID and LQR controller as a hybrid algorithm can enhance the transient and steady-state response characteristics of the system. Implementing the system with PID-LQR ensured stability and smooth response. The use of the PID-LQR technique guaranteed very much rapid response. This means that in practice it will be used to performed more task per operation time than either PID controller or LQR controller. All the control scenarios revealed that the controller was able to supress the link deflection. This means that with any of the controller's design state, the deviation or vibration of the flexible link which could influence the ability of the manipulator to maintain or tracked a referenced position was eliminated in all cases.

KEYWORDS: Controller, Flexible Link Robot Manipulator. PID, PID-LQR, LQR.

INTRODUCTION

Robot can be examined independently as a robot (arm) manipulator, mobile robot, or humanoid. Nevertheless, the study of robot manipulators has attracted large interest in manufacturing industry, education, biomechanics, pipeline monitoring, welding, military, space discovery, hospitals, hotels and online trading (Okubanjo *et al.*, 2017; Ghaleb & Aly, 2018). Robot manipulators are increasingly being used in every aspect of human endeavour, and are actually encountered in nearly every product and have contributed in its production (Baccouch and Dodds, 2020).

Robot manipulators are either rigid or flexible. For increased tensile strength, the design of most robot manipulators is done with steel or aluminium frames thereby resulting in rigid-link robot manipulator that are heavy and immobile. However, advances in material technology have resulted in the development of flexible robot manipulators that are of lightweight, smaller dimensions, portable, improved manoeuvrability, lower power consumption, large work volume, smaller actuators, safer operation, cost effective, reduced control effort, and increased speed of operation as a result of reduced inertia (Ullah *et al.*, 2021). Hence, flexible link robots are increasingly being implemented in several critical applications ranging from space robots and weapon technologies to ground, sea, and air vehicles' control and in systems prone to random actuator failure (Uyulan, 2021).

In the design of efficient and robust control technique, a critical and fundamental step is to accurately compute the dynamic model of a flexible link robot manipulator (FLRM). Rigid assumptions have been made in the design of most industrial robot controllers without considering the dynamics of the actuator (Alam *et al.*, 2018). Hence, the controllers for rigid manipulators are deficient when applied to flexible manipulators and as such exclusive controllers have to be developed to compensate for flexibility in the robot arms (Aziz *et al.*, 2016).

The design and control of flexible robot arm is faced with problem of link flexibility that makes the dynamic modelling relatively difficult compared to rigid link robot arm. The associated inaccuracies due to linear approximation in the dynamic modelling, parameter variations, friction, extremely variation in working conditions, vibrations, change in payload, and external disturbances make the design of controller challenging (Singh *et al*., 2017). Therefore, a controller that can address modelling inaccuracies, parameter variations and effect of external disturbance is required in industrial and space application. The design of suitable controller can provide proper information about the properties of a system and achieve the objectives of the control system (Gupta *et al*., 2021).

This work is designed to study and develop a control system for flexible link robot manipulator that combines two controllers in order to achieve a more robust and efficient approach to enhance trajectory tracking and link deflection suppression.

MATHEMATICAL MODEL

Flexible Link Robot Model

The dynamic behaviour of the flexible link robot manipulator can be expressed in canonical state space representation defined in by:

$$\left.\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx + Du\end{aligned}\right\} \qquad (1)$$

where A = system matrix, B = input matrix, C = output matrix, and D = direct transition matrix. The numerical expressions for A, B, C, and D are given in Alandoli *et al*. (2017):

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K}{J_{eq}} & -\frac{\eta_g K_g^2 \eta_m K_t K_b + B_{eq} R_a}{R_a J_{eq}} & 0 \\ 0 & \frac{K_s(J_{eq}+J_L)}{J_{eq} J_L} & \frac{\eta_g K_g^2 \eta_m K_t K_b + B_{eq} R_a}{R_a J_{eq}} & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{\eta_g K_g^2 \eta_m K_t K_m + R_a}{R_a J_{eq}} \\ -\frac{\eta_g K_g^2 \eta_m K_t K_m + R_a}{R_a J_{eq}} \end{bmatrix},$$

$$C = [1 \quad 1 \quad 0 \quad 0], D = [0]$$

Substituting the values in Table 1 into A and B gives:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 673.07 & -35.1667 & 0 \\ 0 & -1023.07 & 35.1667 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 61.7325 \\ -61.7325 \end{bmatrix}$$

The resulting transfer function model of the FLRM is given by:

$$G(s) = C(sI - A)^{-1}B + D \qquad (2)$$

$$G_{FLRM}(s) = \frac{21606.37499}{s^4 + 35.1667s^3 + 1023.07s^2 + 12308.345s} \qquad (3)$$

Table 1: Simulation parameters (Alandoli *et al.*, 2017)

Definition	Symbol	Value	Unit
Equivalent viscous damping coefficient	B_{eq}	0.004	$Nmrads^{-1}$
Equivalent gear moment of inertia without external load	J_{eq}	0.00208	Kgm^2
Motor efficiency	η_m	0.69	-
Gearbox efficiency	η_g	0.90	-
Back-emf constant	K_b	0.00768	$V/rads^{-1}$
Gear total gearbox ratio	K_g	70	-

Motor armature resistance	R_a	2.6	Ω
Stiffness constant	K_s	1.4	-

PID-LQR Controller

The control system implemented here is hybrid controller that combines Proportional Integral and Derivative (PID) algorithm with Linear Quadratic Regulator (LQR) law. The detail design of the control can be studied in Ekengwu *et al*. (2024) and Okafor *et al*. (2025). The control variable of the PID be u_{PID} to adjust the control law u of the LQR is given by:

$$u_{PID-LQR} = u_{PID}(Kx) = \left[K_p Ke(t) - K_d K\frac{de(t)}{dt} K_i K \int_{t=0}^{t_f} e(t)dt\right]x \quad (4)$$

Or $$u_{PID-LQR} = \left[K_{pk}e(t) - K_{dk}\dot{e} + K_{ik}\int_{t=0}^{t_f} e(t)dt\right]x \quad (5)$$

where $K_{pk} = K_p K$, $K_{dk} = K_d K$, and $K_{ik} = K_d K$. These parameters are optimized by adjusting the gains of PID controller overtime (Ekengwu et al., 2024). The tuned parameters of the PID controller are stated in Table 2. The implemented Q matrix is given as:

$$Q = \begin{bmatrix} Q_{11} & 0 & 0 & 0 \\ 0 & Q_{22} & 0 & 0 \\ 0 & 0 & Q_{33} & 0 \\ 0 & 0 & 0 & Q_{44} \end{bmatrix}$$

Table 2: Tuned Parameters and Performance of the designed PID.

PID parameters	Value
Proportional gain, K_p	2.2027
Integral gain, K_i	1.6119
Derivative gain, K_d	0.075435

RESULTS AND DISCUSSION

In this section, the simulated trajectory response of the designed optimal control system based on hybrid PID-LQR technique for flexible link robot manipulator based on different gain matrices of the LQR with respect to the tuned Q matrix. Thus, these scenarios are for PID-LQR1, PID-LQR2, and PID-LQR3. It should be noted that the final values of the tuned PID parameters in the hybrid PID-LQR controller were: $K_p = 2.2$, $K_i = 0.12$, and $K_d = 0.05$.

4Response of First Designed PID-LQR

Figure 1 shows the simulated output response of PID-LQR for $K = [7.4162 \; -1.3451 \;\; 0.4714 \;\; 0.2611]$ using step input signal and sinusoidal input signal. In Figure 1a, it is obvious that the flexible link robot manipulator controlled by the PID-LQR accurately tracks the reference input with rise time of 0.1444 s and overshoots the reference input by 0.5119% with little settling time of 0.2637 s. The tracking performance of the system when a sinusoidal signal is applied as the reference trajectory is shown in Figure 1b. The figure shows that the system followed the sinusoidal waveform of the input with rise time of 0.3587 s.

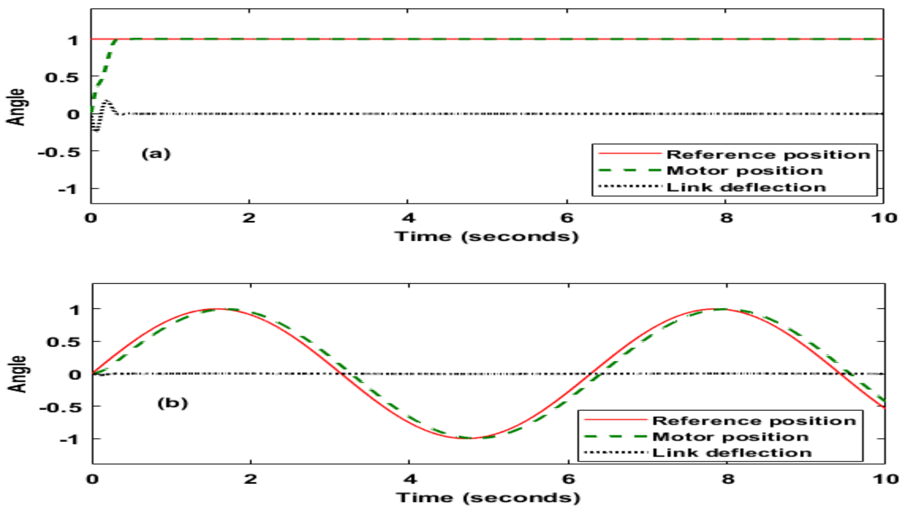

Figure 1: Trajectory tracking of PID-LQR controlled FLRM.

In the first case with (a) step input (b) sinusoidal input

In Figure 1a, the peak value of the link deflection is 0.2436 at time t = 0.0706 s prior to the transition to zero or no link deflection steady-state at time t = 0.5163 s. Looking at Figure 1b, it is clear that with PID-LQR controlled system in this case, an improved trajectory following of periodic input signal is achieved.

Response of Second Designed PID-LQR

With matrix gain of the LQR changed to $K = [7.4162 \; -1.3451 \;\; 0.4714 \;\; 0.2611]$ and PID parameters used to further optimize the gains, the simulated trajectory tracking performance of the system for step input signal and sinusoidal input signal is shown in Figure 2.

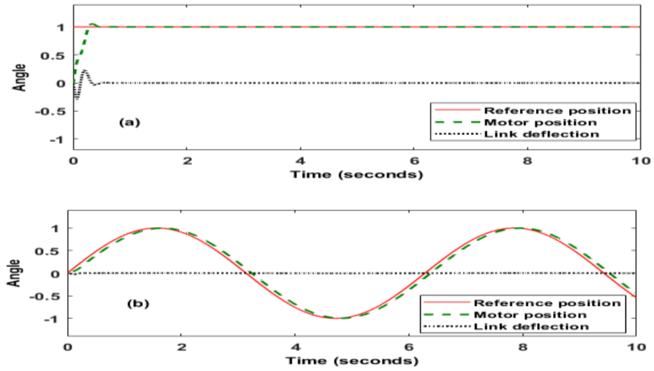

Figure 2: Trajectory tracking of PID-LQR controlled FLRM.

In the second case with (a) step input (b) sinusoidal input

The output of the system as shown in Figure 2a reveals that it accurately followed the reference unit step input signal with improved rise time of 0.1170 s and settling time of 0.43611 s, but with significant overshoot of 9.8129%. This performance indicates that the designed PID-LQR controller in this case will provide faster response than the previous case in the first design However, it showed deterioration in settling time and overshoot performance. The control system in this case also ensured that the deflection in flexible link is suppressed. The peak magnitude of the link deflection was 0.2914 at t = 0.0706 s while the transition time to zero or no deflection was 0.5527 s.

Response of Third Designed PID-LQR

In this case, the PID parameters were used to adjust the LQR matrix gain $K = [10.000 \;\; -2.4970 \;\; 0.6195 \;\; 0.3352]$ to further examine the performance of the designed PID-LQR controller. Figure 3 shows the response of the designed control system as it follows a step input signal (Figure 3a) and unit sinusoidal input waveform (Figure 3b).

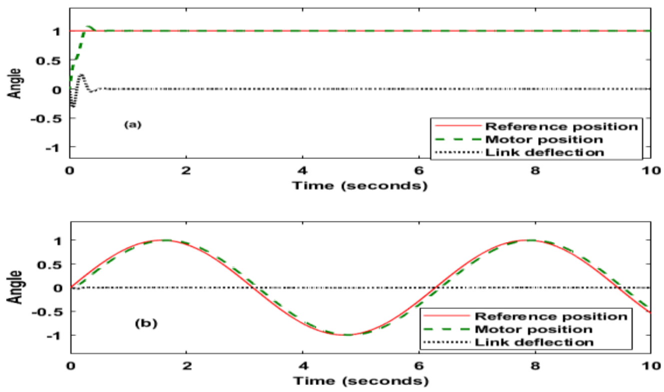

Figure 3: Trajectory tracking of PID-LQR controlled FLRM.

In the third case with (a) step input (b) sinusoidal input

As shown in Figure 3a, it is evident that system followed the unit step input with very much improved rise time of 0.1096 s compared with the previous cases. However, the system shows deterioration in settling time, 0.4859 s and overshoots the reference input with peak percentage of 14.8344. On the other hand, the system showed a link deflection of peak value: 0.3126 at time t = 0.0636 s with a transient time of 0.6240 s. The tracking performance of the system when the referenced trajectory to be tracked is a sine wave signal is shown in Figure 3b. It indicates that the system was able to follow the input waveform signal with a response (rise) time of 0.3882 s.

The comparison plots of the PID-LQR controller for the three varied gains of K matrix are shown in Figure 4a and b for step input signal and sinusoidal waveform input signal respectively.

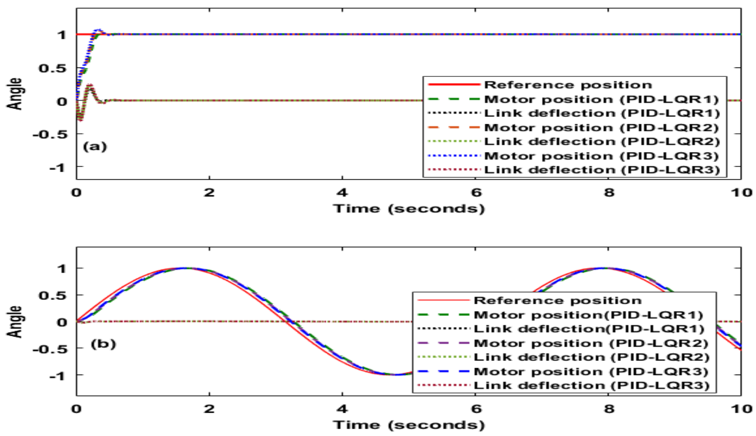

Figure 4: Trajectory tracking of PID-LQR controlled FLRM.

For different cases with (a) step input (b) sinusoidal input

Figure 4a shows that the PID-LQR yielded the fastest trajectory tracking of the motor position for unit step input signal in the third case (PID-LQR3), but has the worst overshoot. Also, for the link deflection analysis, simulated output showed that each of the PID-LQR controls: PID-LQR1, PID-LQR2, and PID-LQR3 yield peak deflection of 0.2436, 0.2914 at time t = 0.0706 s and 0.3126 at t = 0.0636 s, respectively. In each case, the time for the link deflection to transit from the peak value (i.e. its highest deflection point) to zero is 0.5163 s for PID-LQR1, 0.5527 s for PID-LQR2, and 0.6240 s for PID-LQR3, respectively. In Figure 4b, the waveform of the periodic input signal revealed that the designed PID-LQR control system achieved improved path or trajectory tracking performance in all cases for a given periodic time varying positioning operation.

CONCLUSION

Three different LQR algorithms were designed in terms of the feedback matrix gain by varying the principal diagonal elements Q_{11} and Q_{22} of the Q weighting matrix. Transfer function model of the system was determined from the state space equation using appropriate MATLAB syntax. A PID controller was developed and combined with LQR to achieved the so called PID-LQR. The developed PID-LQR was introduced into the closed-loop control system of the FRLM. Simulations were conducted in terms of step input signal and sinusoidal input signal for varied Q matrix of the LQR. The results showed that the PID-LQR controller offered promising time responses and were able to eliminate link deflection at different cases considered with respect to varied update law given by the gain or K matrix of the LQR. That is, the controller ensured that desired position or trajectory is tracked and at the same time guaranteed zero deflection of the link was achieved in all cases.

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