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APPLICATION OF GRAPH COLOURING TECHNIQUES IN MOBILE NETWORKS

***Dogondaji A. M. and Attahiru M. B.**

Department of Mathematics, Usmanu Danfodiyo University, Sokoto-Nigeria.

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***Corresponding Author: Dogondaji A. M.**
Department of Mathematics, Usmanu Danfodiyo University, Sokoto-Nigeria.

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ABSTRACT

Graph colouring is an essential tool in graph theory with extensive application in communication and network optimization. This paper investigates the various concept of graph colouring in mobile network system. The research proposed model mobile network cells which are represented at vertices of the graph while edges indicate interference between the adjacent cells. The work assign distinct colours to adjacent vertices such that no two neighboring cells share the same frequency. The chromatic number optimization is used to achieve minimal frequency allocation. The research also highlights on the importance of graph colouring as both theoretical and practical techniques in solving real-world communication problems. The research discusses some relevance of chromatic number determination in estimating the minimum number of frequencies required for interference free communication. Furthermore, it serves as a practical frame work for efficient channel assignment and network planning in modern mobile communication system.

KEYWORDS: chromatic number, graph coloring, graph structure, network system.

1:1 INTRODUCTION

Graph theory a vital area within discrete mathematics which concerned with the study of graph mathematics structure used to model pair wise relationships among objects Kubale, 2004). One of the most significant and widely applied concept within graph theory is graph coloring, which refers to assigning colors to the element of a graph (typically vertices) under specific constraints(Jensen,2011). (Dogondaji and Attahiru, 2025). The most common form of vertex coloring involves assigning colours to each vertex such that no two adjacent vertices share the same colors. The minimum number of colours needed to achieve this is known as the chromatic number of graph (Bondy and Murty,2008) . Graph coloring problems have important applications in many real-life scenarios, including scheduling, register allocation in compilers, map colouring frequency assignment in mobile networks,and task assignment in distributed system (Garey and Johnson, 1979) and (Ghosh, Das and Sen(2010).

Historically, one of the most notable graph colouring problems is the four colour theorem which asserts that any planar map can be coloured using not more than four distinct colours in such a way that no two adjacent regions have the same colours. This problem was solved by Hale,1980) and (Appel and Haken, 1977) using computer aided proof thereby making a significant milestone in both discrete mathematics and computer science (Graggs,1992).

1.2 DEFINITIONS OF SOME BASIC TERMS

Definition 1.2.1

A graph is a mathematical structure consisting of a set of vertices (ornodes) and a set of edges that connect pairs of vertices.

Definition 1.2.2

A vertex is a fundamental unit represented as a point in a graph the plural is vertices.

Definition 1.2.3

An edge is a line that connects two vertices in a graph if there is an edgebetween vertex a and vertex b, we say a and b are adjacent.

Definition 1.2.4

Adjacent vertices:Two vertices are adjacent if they are connected directly by an edge.

Definition 1.2.5

Degree of a vertex:The degree of a vertex is the number of edges connected to it.

Definition 1.2.6

Path**:** a path is a sequence of edges that connects a sequence of distinctvertices.

Definition 1.2.7

Cycle:A cycle is a path that starts and ends at the same vertex, with allintermediate vertices being distinct.

Definition 1.2.8

Chromatic number**:** The chromatic number of a graph is the minimum number of colours5 needed to colour the vertices of the graph such that no two adjacentvertices share the same colour(Raniwale and Chiuec (2005).

Definition 1.2.9

Graph colouring**:** Graph colouring refers to the assignment of colours to the elements of agraph (usually vertices) such that adjacent elements do not share thesame colour.

Definition 1.2.10

Vertex colouring: in vertex colouring, each vertex is assigned a colour in such a way that no two adjacent vertices have the same colour.

Definition 1.2.11

Edge colouring: Each edge is coloured so that no two edges sharing a common vertex and have the same colour.

Definition 1.2.12

Planar graph: a planar graph is a graph that can be drawn on a plane without any edges crossing.

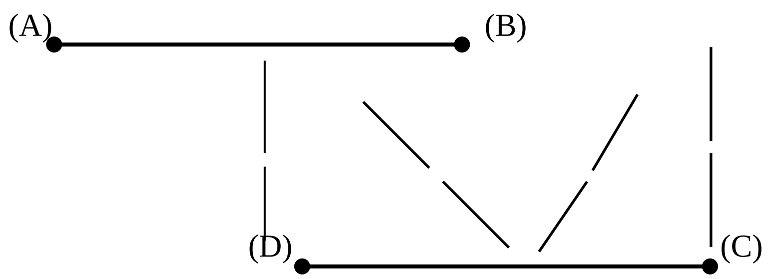
Definition 1.2.13

Bipartite graph: a bipartite graph is a graph whose vertices can be divided into two disjoint sets such that no two vertices within the same set are adjacent.

2:0 METHODOLOGY

In this chapter, the research will consider the various method of formation of graph colouring. It begins as follows:

Example 2.1. Lets consider the following graph structures:



Fig(a) Assigning Colours

Let the colours to be assign be represented as: A: Red, B: Green, C: Red, D: Blue

Above is a complete graph with 3 vertices and each is connected to one another

Vertices: A,B,C

Edges: AB, AC, BC

colouring : Each vertex must have a unique colour since all are adjacent to each other

A: Colour1, B: Colour 2, C: colour 3

The Chromatic number is now represented as $x(k_3)$

Example 2:2 Bipartite graph K_2

Description : The vertices are divided into two sets: every vertex in one set is connects to the other set and are represented as follows:

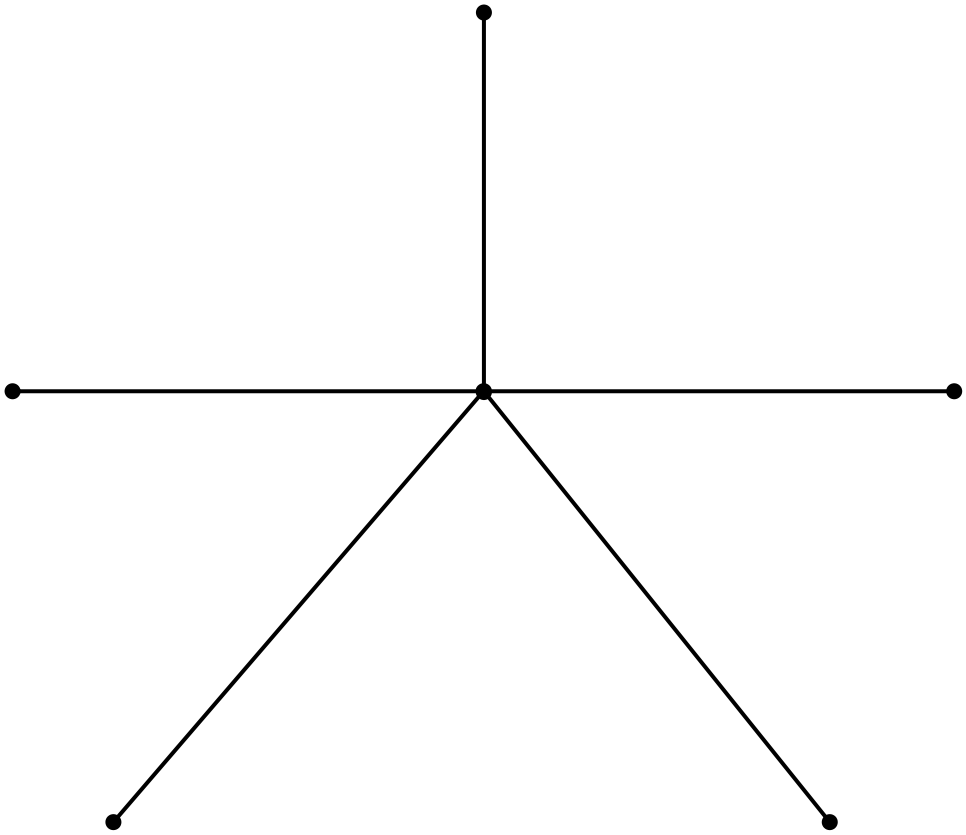
Set 1: A, B

Set 2: C, D, E

Colouring : No edges within a sets

A,B: Colour 1

C,D,E: Colour 2



Chromatic

Number is : 2

Fig (b) Bibatite graph

Description : one central node is connected to five outer nodes.

Central: A

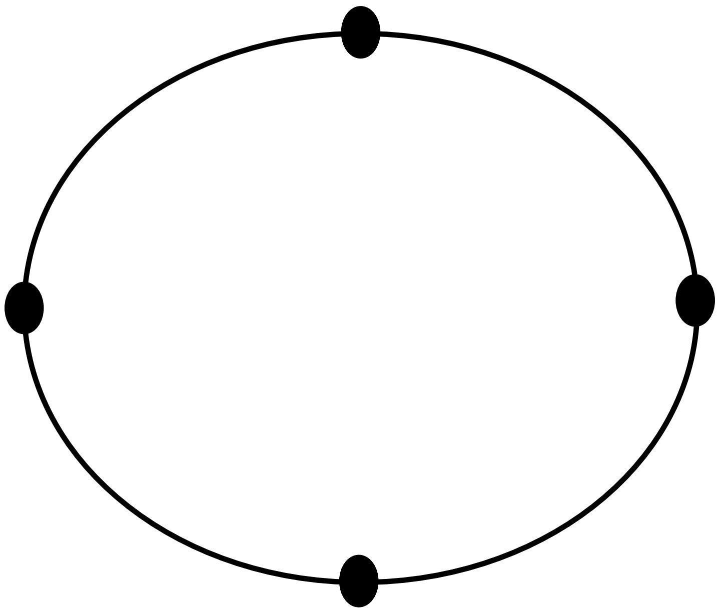
Outer: B,C,D,E,F

Colouring: A: Colour 1, B,F: Colour 2

Chromatic number: 2

Application: Used in network hub with spokes in wireless network interference avoidance.

Example 2: 3 **cycle graph** $X(4)$**:**



Fig(c) Cycle graph

Description : four vertices in a cycle (square shape)

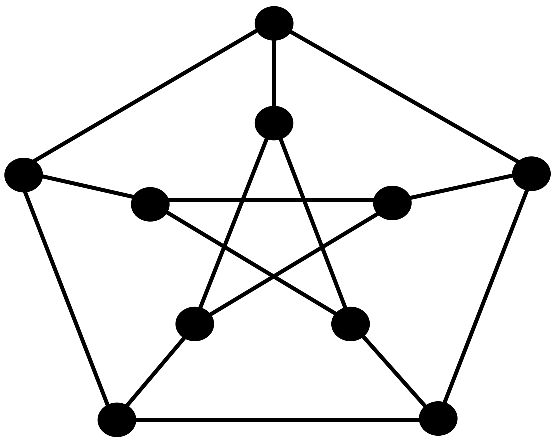
A-B-C-D-A

Colouring: A,C: Colour 1 and B,D: Colour 2

Chromatic number $X(2)$: 2 (since it's even)

Application: ring networks or round -robin scheduling.

Example 2:4 Petersen graph



Fig(d) petersen graph

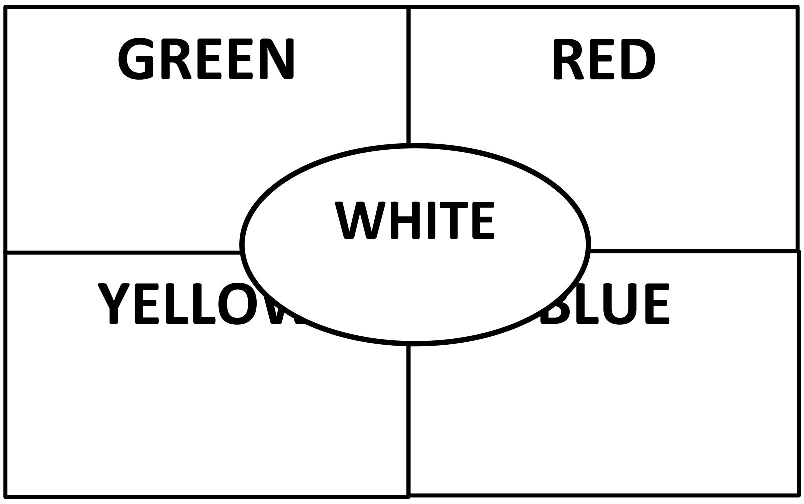
Description: A complex non planer graph with 10 vertices with its edges colouring

results: These can be coloured using 3 colours

And the Chromatic number: 3

Application: In error detection codes theoretical counter examples in graph theory.

Example 2.5 planer map colouring



Fig(e) planer map colouring

Description: A map with 5 regions touching each other.

Using the four colour theorem,we can colour any planer map using almost 5 colours.

Colouring will be as follows:

Region A: colour 1

Region B: colour 2

Region C: colour 3

Region D: colour 4

Region E: colour 5

Chromatic number will be $X(5)$.

4.0 RESULTS AND DISCUSSIONS

4.1 Class Timetable Scheduling

Description: each class has a vertex edges between two class of colours if they share similar graph formed based on conflict matrix

Colouring: assign colours to time slots so that no conflicting classes occur at the same time. The Chromatic number depends on class conflicts.

Application: applied in automated time table generation in schools/universities

4.2 Register Allocation (Compiler Design)

Description: variables are represented as vertices and edges represent variables

Colouring: Assign registers (colours) to variable without conflict.

Chromatic number: minimum number of registers needed.

Application: optimizing machine level code performance.

4.3 Soduko Puzzles as Graph Colouring

Description: each cell is a vertex, edges between any two cells in the same row/column, or 3×3 box. Colouring: assign digit (1-9) such that no two connected vertices have the same number.

Chromatic number: 9

Application: game design, constraint satisfaction problems.

4.4 frequency assignment in cellular networks

Description: each tower is a vertex towers close to each other as connected by edges colouring: assign frequencies (colours) so that no two nearby towers share the same frequency.

chromatic number: depends on density of towers.

application: tele-communications, mobile networks.

4.5 vertex colouring

Description: vertex colouring is the process of assigning different colours to each vertex of a graph in such a way that no two adjacent vertices (connected by an edge) share the same colour. each vertex can represent a base station, device, or cell in a network system.

colouring: if two vertices are connected, they must have different colours. example, in a triangular graph (three vertices all are connected), we need three distinct colours as follows:

vertex a → colour 1

vertex b → colour 2

vertex c → colour 3

chromatic number: 3

the chromatic number of this graph is 3, because at least three colours are needed to colour all vertices without conflict.

vertex colouring is applied in frequency assignment problems. each base station (vertex) is assigned a frequency (colour) so that neighbouring stations using the same channel don't cause interference. this ensures efficient use of the frequency spectrum and improves signal quality.

4.6 Edge colouring

D escription: edge colouring assigns colours to the edges of a graph so that no two edges sharing the same vertex have the same colour. edges can represent communication links between mobile devices or transmitters. colouring: for a triangle graph with vertices a, b, and c, the edges (a–b), (b–c), and (c–a) must all have different colours.

edge a–b → colour 1

edge b–c → colour 2

edge c–a → colour 3

chromatic number (edge chromatic number):

for a triangle, the edge chromatic number is 3.

applied in mobile networks:

edge colouring is used in channel allocation, where each edge represents a communication link. assigning different colours (channels) ensures that no two transmissions connected to the same node interfere with one another, improving network performance.

4.7 Face (or Region) Colouring

Description: Face or region colouring is applied to planar graphs (graphs that can be drawn on a plane without edges crossing). Each region (face) of the graph is assigned a colour so that no two adjacent regions share the same color.

CONCLUSION

This research has demonstrated a powerful mathematical framework for solving frequency allocation and interference management problem in mobile communication network systems. This is done by represent cellular regions as vertices and interference relationships as edges. The coloring of the resulting graphs shows that no two adjacent cells share the same frequency. The research also helps in minimizing co-channel interference and improving overall communication quality in networks.

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