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A PRELIMINARY ANALYSIS OF THE ROUTE OF THE UNSOLVABLE TRAVELING SALESMAN PROBLEM WITH PRIME, NATURAL, INTEGER, REAL NUMBERS AND FACTORIALS

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SUMMARY

The intention of this article is to enable a relevant research method for the traveling salesman problem using prime numbers. Initially, a preliminary analysis was conducted to be transformed into research based on two other relevant existing articles cited in the bibliography, in order to expand Number Theory and Computation Theory to...this complex problemgiven that **Traveling salesman problem** It is classified as NP-hard in its optimization version and NP-complete in its decision version, meaning that there is no known efficient algorithm to solve it in polynomial time for any number of cities.One way to ratify and show an outcome of the bibliographic reference article [1].

KEYWORDS: Traveling Salesman, NP-Hard, NP-Complete, Prime Numbers, Exponential, Factorial, Combinatorial Analysis.

INTRODUCTION

This article presents a Fortran program on Linux Ubuntu designed to analyze the Traveling Salesman Route using decomposition and the study of prime numbers with factorial problems, noting thatIt can be extended to all types of numbers in the field of mathematics. The program pIt works for 10 cities, but to significantly increase the number of cities, it's better to change the algorithm to...**Write the routes directly in the TXT file.**avoiding storing them all in RAM. If you want**Display the results on the screen and simultaneously save them to a TXT file.** It can also be done in Fortran usingwrite(*,...)andwrite(10,...)for each result. This program now shows**all distances**, the number of routes for each distance and, most importantly,**all routes that result in a tie when the distance appears more than once**.Knowing that the **Traveling salesman problem**It is classified as NP-hard in its optimization version and NP-complete in its decision version, meaning that there is no known efficient algorithm to solve it in polynomial time for any number of cities.With the existence of several attempts at in-depth and computational analysis of the same problem.

Theoretical Framework:

Factorials written as the sum of powers of 2

Similar to Table 3 in the article. Each value ofn!it is written as a**exact sum of powers of 2**Starting with the highest power. This is the canonical binary expansion, therefore each power appears at most once.

Number	Factorial	Value of n!	Sum of powers of 2
0	0!	1	1
1	1!	1	1
2	2!	2	2
3	3!	6	2^2 + 2
4	4!	24	2^4 + 2^3
5	5!	120	2^6 + 2^5 + 2^4 + 2^3
6	6!	720	2^9 + 2^7 + 2^6 + 2^4
7	7!	5.040	2^12 + 2^9 + 2^8 + 2^7 + 2^5 + 2^4
8	8!	40.320	2^15 + 2^12 + 2^11 + 2^10 + 2^8 + 2^7
9	9!	362,880	2^18 + 2^16 + 2^15 + 2^11 + 2^8 + 2^7

Number	Factorial	Value of n!	Sum of powers of 2
10	10!	3,628,800	2^21 + 2^20 + 2^18 + 2^17 + 2^16 + 2^14 + 2^12 + 2^11 + 2^10 + 2^9 + 2^8
11	11!	39,916,800	2^25 + 2^22 + 2^21 + 2^16 + 2^12 + 2^10 + 2^8
12	12!	479.001.600	2^28 + 2^27 + 2^26 + 2^23 + 2^19 + 2^18 + 2^15 + 2^14 + 2^13 + 2^12 + 2^11 + 2^10
13	13!	6,227,020,800	2^32 + 2^30 + 2^29 + 2^28 + 2^25 + 2^24 + 2^21 + 2^19 + 2^15 + 2^14 + 2^11 + 2^10
14	14!	87.178.291.200	2^36 + 2^34 + 2^30 + 2^27 + 2^26 + 2^21 + 2^20 + 2^19 + 2^17 + 2^16 + 2^13 + 2^11

15	15!	1,307,674,368,000	2^40 + 2^37 + 2^36 + 2^30 + 2^29 + 2^28 + 2^26 + 2^25 + 2^24 + 2^22 + 2^21 + 2^20 + 2^18 + 2^17 + 2^16 + 2^14 + 2^12 + 2^11
16	16!	20,922,789,888,000	2^44 + 2^41 + 2^40 + 2^34 + 2^33 + 2^32 + 2^30 + 2^29 + 2^28 + 2^26 + 2^25 + 2^24 + 2^22 + 2^21 + 2^20 + 2^18 + 2^16 + 2^15
17	17!	355,687,428,096,000	2^48 + 2^46 + 2^41 + 2^40 + 2^38 + 2^37 + 2^36 + 2^35 + 2^34 + 2^33 + 2^31 + 2^30 + 2^29 + 2^27 + 2^26 + 2^25 + 2^23 + 2^22 + 2^19 + 2^18 + 2^16 + 2^15
18	18!	6,402,373,705,728,000	2^52 + 2^50 + 2^49 + 2^47 + 2^45 + 2^44 + 2^43 + 2^42 + 2^41 + 2^39 + 2^38 + 2^37 + 2^35 + 2^34 + 2^31 + 2^30 + 2^27 + 2^25 + 2^22 + 2^21 + 2^20 + 2^17 + 2^16

Number	Factorial	Value of n!	Sum of powers of 2
19	19!	121,645,100,408,832,000	2^56 + 2^55 + 2^53 + 2^52 + 2^45 + 2^43 + 2^41 + 2^40 + 2^39 + 2^36 + 2^33 + 2^32 + 2^26 + 2^25 + 2^23 + 2^19 + 2^16
20	20!	2,432,902,008,176,640,000	2^61 + 2^56 + 2^55 + 2^54 + 2^49 + 2^48 + 2^46 + 2^45 + 2^42 + 2^41 + 2^40 + 2^38 + 2^37 + 2^36 + 2^35 + 2^34 + 2^31 + 2^25 + 2^23 + 2^21 + 2^20 + 2^18

Observation: 0! = 1and1! = 1The form2^0it was written simply as1Unlike some examples in the article, the expressions above have been checked so that each sum is exactly equal to its respective factorial.

Factorials as the sum of powers of 2 with prime exponents.

Table organized according to the indicated sequence: the first term of each row is2^e, with the main exponents2, 4, 5, 7, 11, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 53, 59, 61The remaining terms complete the sum exactly up to the value ofn!, always in descending order of exponents when representation is possible.

Number	Factorial	Value of n!	Requested sum
3	3!	6	2^2 + 2^1
4	4!	24	2^4 + 2^3
5	5!	120	2^5 + 2^6 + 2^4 + 2^3
6	6!	720	2^7 + 2^9 + 2^6 + 2^4
7	7!	5.040	2^11 + 2^11 + 2^9 + 2^8 + 2^7 + 2^5 + 2^4
8	8!	40.320	2^11 + 2^15 + 2^12 + 2^10 + 2^8 + 2^7
9	9!	362,880	2^13 + 2^18 + 2^16 + 2^14 + 2^13 + 2^11 + 2^8 + 2^7

Number	Factorial	Value of n!	Requested sum
10	10!	3,628,800	2^17 + 2^21 + 2^20 + 2^18 + 2^16 + 2^14 + 2^12 + 2^11 + 2^10 + 2^9 + 2^8
11	11!	39,916,800	2^19 + 2^25 + 2^22 + 2^20 + 2^19 + 2^16 + 2^12 + 2^10 + 2^8
12	12!	479.001.600	2^23 + 2^28 + 2^27 + 2^26 + 2^19 + 2^18 + 2^15 + 2^14 + 2^13 + 2^12 + 2^11 + 2^10
13	13!	6,227,020,800	2^29 + 2^32 + 2^30 + 2^28 + 2^25 + 2^24 + 2^21 + 2^19 + 2^15 + 2^14 + 2^11 + 2^10
14	14!	87.178.291.200	2^31 + 2^36 + 2^33 + 2^32 + 2^31 + 2^30 + 2^27 + 2^26 + 2^21 + 2^20 + 2^19 + 2^17 + 2^16 + 2^13 + 2^11
15	15!	1,307,674,368,000	2^37 + 2^40 + 2^36 + 2^30 + 2^29 + 2^28 + 2^26 + 2^25 + 2^24 + 2^22 + 2^21 + 2^20 + 2^18 + 2^17 + 2^16 + 2^14 + 2^12 + 2^11
			2^41 + 2^44 + 2^40 +

16	16!	20,922,789,888,000	2^34 + 2^33 + 2^32 + 2^30 + 2^29 + 2^28 + 2^26 + 2^25 + 2^24 + 2^22 + 2^21 + 2^20 + 2^18 + 2^16 + 2^15
17	17!	355,687,428,096,000	2^43 + 2^48 + 2^45 + 2^44 + 2^43 + 2^41 + 2^40 + 2^38 + 2^37 + 2^36 + 2^35 + 2^34 + 2^33 + 2^31 + 2^30 + 2^29 + 2^27 + 2^26 + 2^25 + 2^23 + 2^22 + 2^19 + 2^18 + 2^16 + 2^15
18	18!	6,402,373,705,728,000	Impossible: 2^53 = 9,007,199,254,740,992 > 6,402,373,705,728,000
19	19!	121,645,100,408,832,000	Impossible: 2^59 = 576,460,752,303,423,488 > 121,645,100,408,832,000

Number	**Factorial**	**Value of n!**	**Requested sum**
20	20!	2,432,902,008,176,640,000	2^61 + 2^56 + 2^55 + 2^54 + 2^49 + 2^48 + 2^46 + 2^45 + 2^42 + 2^41 + 2^40 + 2^38 + 2^37 + 2^36 + 2^35 + 2^34 + 2^31 + 2^25 + 2^23 + 2^21 + 2^20 + 2^18

Important note about18!: There is no sum of positive terms starting with2^53, why2^53 = 9,007,199,254,740,992is bigger than18! = 6,402,373,705,728,000To obtain a positive sum for18!, the initial exponent would have to be at most52.

Factorial table in base 2 and result in prime numbers.

Number	**Factorial in base 2**	**Result in Prime Numbers**
3	(2+1)(2)(1)	2^2+2
4	(2+2)(2+1)(2)(1)	(2^4)+2^3
5	(2+2+1)(2+2)(2+1)(2)(1)	2^5 + 3 * 2^4 + 2 * 2^3
6	(2+2+2)(2+2+1)(2+2)(2+1)(2)(1)	3·2^7+9·2^5+3·2^4
7	(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)(1)	2·2^11+7·2^7+1·2^5+1·2^4
8	(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	19·2^11+2^10+2^8+2^7
9	(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	44·2^13+2^11+2^8+2^7
	(2+2+2+2)(2+2+2+1)(2+2+2)	

10	(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	27·2^17+2^16+2^14+2^12+2^11+2^10+2^9+2^8
11	(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	76·2^19+2^16+2^12+2^10+2^8
12	(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	57·2^23+2^19+2^18+2^15+2^14+2^13+2^12+2^11+2^10
Number	**Factorial in base 2**	**Result in Prime Numbers**
13	(2+2+2+2+2+1)(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	11·2^29+2^28+2^25+2^24+2^21+2^19+2^15+2^14+2^11+2^10
14	(2+2+2+2+2+2)(2+2+2+2+2+2+1)(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2)(2+1)(2)1	40·2^31+2^30+2^27+2^26+2^21+2^20+2^19+2^17+2^16+2^13+2^11
15	(2+2+2+2+2+2+1)(2+2+2+2+2+2)(2+2+2+2+2+1)(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	9·2^37+2^36+2^30+2^29+2^28+2^26+2^25+2^24+2^22+2^21+2^20+2^18+2^17+2^16+2^14+2^12+2^11
16	(2+2+2+2)(2+2+2+2+2+2+1)(2+2+2+2+2+2)(2+2+2+2+2+1)(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+1)(2)1	9·2^41+2^40+2^34+2^33+2^32+2^30+2^29+2^28+2^26+2^25+2^24+2^22+2^21+2^20+2^18+2^16+2^15
17	(2+2+2+2+1)(2+2+2+2)(2+2+2+2+2+2+2+1)(2+2+2+2+2+2)(2+2+2+2+2+1)(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	40·2^43+2^41+2^40+2^38+2^37+2^36+2^35+2^34+2^33+2^31+2^30+2^29+2^27+2^26+2^25+2^23+2^22+2^19+2^18+2^16+2^15
18	(2+2+2+2+2)(2+2+2+2+1)(2+2+2+2)(2+2+2+2+2+2+1)	

	(2+2+2+2+2+2)(2+2+2+2+2+1) (2+2+2+2+2)(2+2+2+2+1) (2+2+2+2)(2+2+2+1)(2+2+2) (2+2+1)(2+1)(2)1	Impossible: 2^53 > 18!
19	(2+2+2+2+2+1)(2+2+2+2+2+2) (2+2+2+2+2+1)(2+2+2+2) (2+2+2+2+2+2+1)(2+2+2+2+2+2) (2+2+2+2+2+1)(2+2+2+2+2) (2+2+2+2+1)(2+2+2+2)(2+2+2+1) (2+2+2)(2+1)(2)1	Impossible: 2^59 > 19!
20	(2+2+2+2+2+2)(2+2+2+2+2+1) (2+2+2+2+2)(2+2+2+2+1) (2+2+2+2)(2+2+2+2+2+2+1) (2+2+2+2+2+2)(2+2+2+2+2+1) (2+2+2+2+2)(2+2+2+2+1)(2+2+2) (2+2+2+1)(2+2)(2+1)(2)1	2^61+2^56+2^55+2^54+2^49+2^48 +2^46+2^45+2^42+2^41+2^40+2^3 8+2^37+2^36+2^35+2^34+2^31+2^ 25+2^23+2^21+2^20+2^18

Notice: The first five results were kept exactly as provided in the model. The expression of the article for5!The sum is 96, not 120; therefore, it was preserved as a reference forformat, but it must be corrected for an exact match. For18!and19!, the main exponents2^53and2^59They are larger than the factorials themselves, making a positive sum starting with those terms impossible.

DISCUSSION

What does this version do?

Now the program produces**two levels of information**.

1. All distances found:

DISTANCENUMBER OF ROUTES

914

928

9312

9420

9536

2. All routes that are tied at each distance:

DISTANCE = 91-> 4 ROUTES

1 : 1 -> 4 -> 3 -> ... -> 1

2 : 1 -> 5 -> 7 -> ... -> 1

3 : 1 -> 8 -> 2 -> ... -> 1

4 : 1 -> 9 -> 6 -> ... -> 1

DISTANCE = 92-> 8 ROUTES

5 : 1 -> 3 -> 7 -> ... -> 1

6 : 1 -> 6 -> 4 -> ... -> 1

...

This way you can study the**complete distribution of TSP solutions**and not just the optimal solution.

ANDIn each row, the coefficients must consecutively follow the sequence of primes: 2, 3, 5, 7, 11,...

But there's a missing rule for exponents. Furthermore, if each prime can only appear once, some lines have no solution. For example, for 3!=6:

$$2 \cdot 2^a + 3 \cdot 2^b = 6$$

It has no solution when a, b are non-negative integers.

THEtable using the most practical rule:

$$n! = 2 \cdot 2^a + 3 \cdot 2^b + 5 \cdot 2^c + 7 \cdot 2^d + \cdots$$

allowing that:

- the prime numbers are used in the order 2, 3, 5, 7, ...;
- a cousin can repeat when necessary;
- the last term is adjusted so that the sum is exactly equal to an!.

For example:

$$3! = 2 \cdot 2^1 + 2 = 6$$

$$4! = 2 \cdot 2^3 + 2 \cdot 2^2 = 24$$

$$5! = 2 \cdot 2^5 + 2 \cdot 2^4 + 2 \cdot 2^3 + 2 = 120$$

This rule can be used to generate the complete table up to 20!

==

Traveling Salesman Problem(Program Output File in the Appendix)

==

NUMBER OF CITIES:10

Routes analyzed:362880

==

ALL DISTANCES FOUND

==

DISTANCENUMBER OF ROUTES

1042

1052

1062

1076

10816

10924

11024

11146

11254

11366

11496

115118

116228

117256

118382

119440

120524

121672

122828

1231034

1241306

1251444

1261850

1272078

1282372

1292676

1303232

1313452

1324216

1334370

1345152

1355424

1366264

1376644

1387410

1397508

1408326

1418520

1429526

1439740

14410502

14510464

14611198

14711062

14811440

14911138

15011762

15111496

15212002

15311304

15411634

15510996

15610796

15710038

15810152

1599416

1609418

1618554

1628220

1637164

1646930

1655774

1665438

1674738

1684458

1693566

1703548

1713020

1722500

1732088

1741738

1751234

176916

177648

178494

179304

180186

181116

18264

18320

1848

1854

1862

TIED ROUTES

===

DISTANCE =104->2 ROUTES

34928 :1 ->2 ->9 ->10 ->8 ->6 ->5 ->7 ->4 ->3 ->1

47531 :1 ->3 ->4 ->7 ->5 ->6 ->8 ->10 ->9 ->2 ->1

- -
DISTANCE =105->2 ROUTES
- -
266:1 ->2 ->3 ->4 ->7 ->5 ->6 ->8 ->10 ->9 ->1

320522 :1 ->9 ->10 ->8 ->6 ->5 ->7 ->4 ->3 ->2 ->1

- -
DISTANCE =106->2 ROUTES
- -
34568 :1 ->2 ->9 ->10 ->5 ->6 ->8 ->7 ->4 ->3 ->1

47891 :1 ->3 ->4 ->7 ->8 ->6 ->5 ->10 ->9 ->2 ->1

- -
DISTANCE =107->6 ROUTES
- -
296:1 ->2 ->3 ->4 ->7 ->8 ->6 ->5 ->10 ->9 ->1

34427 :1 ->2 ->9 ->3 ->4 ->10 ->8 ->6 ->5 ->7 ->1

35192 :1 ->2 ->9 ->10 ->4 ->7 ->8 ->6 ->5 ->3 ->1

52229 :1 ->3 ->5 ->6 ->8 ->7 ->4 ->10 ->9 ->2 ->1

213595 :1 ->7 ->5 ->6 ->8 ->10 ->4 ->3 ->9 ->2 ->1

318410 :1 ->9 ->10 ->5 ->6 ->8 ->7 ->4 ->3 ->2 ->1

- -
DISTANCE =108->16 ROUTES
- -
…

How do the exponents from 2^0 to 2^20 work?they are:

Exponent	Power	Value
0	2^0	1
Exponent	**Power**	**Value**
1	2^1	2
2	2^2	4
3	2^3	8
4	2^4	16
5	2^5	32
6	2^6	64
7	2^7	128
8	2^8	256
9	2^9	512
10	2^10	1.024
11	2^11	2.048
12	2^12	4.096
13	2^13	8.192
14	2^14	16,384
15	2^15	32,768
16	2^16	65,536
17	2^17	131.072
18	2^18	262,144
19	2^19	524,288
20	2^20	1,048,576

10	(2+2+2+2)(2+2+2+1)(2+2+2)(2+2+2+1)(2+2+2)(2+2+1)(2+2)(2+1)(2)1	27·2^17+2^16+2^14+2^12+2^11+2^10+2^9+2^8

Sum of tied routes

- **Number of distances/groups:**83
- **Combined routes:**362,880
- **Routes in groups with a tie:**362,880
- **Pairs of tied routes:**1,539,832,276

The principal sum is2 + 2 + 2 + 6 + ... + 2 = 362,880Since all the quantities presented are greater than 1, the 362,880 routes belong to groups with at least two routes of the same distance.

$$27 \times 2^{17} = 27 \times 131.072$$

$$\boxed{27 \times 2^{17} = 3.538.944}$$

As:

$$\boxed{10! = 3.538.944 + 89.856 = 3.628.800}$$

To relate 2^{17} to the number of routes: 2^{17}=131.072

In the expression 10!:

10! = 27.2^17 + remainder we have:

27.2^{17}=27.131.072=3.538.944

As:

10! = 3,628,800

The rest is:

3,628,800 - 3,538,944 = 89,856

Like this:

$$\boxed{10! = 3.538.944 + 89.856}$$

Relating to the routes and distances in the table:

Quantity	Value
2^{17}	131,072 routes
$27 \cdot 2^{17}$	3,538,944 routes
Total number of routes, $10!$	3,628,800 routes
Average distance	150
Total sum of distances	54,432,000

If each group of 2^{17} routes were associated with an average distance of 150, then:

$$2^{17} \cdot 150 = 131.072 \cdot 150 = 19.660.800$$

Therefore,

$$27 \cdot 2^{17} \cdot 150 = 3.538.944 \cdot 150 = 530.841.600$$

Therefore, 2^17 represents 131,072 routes; multiplied by 27, it represents 3,538,944 routes. Using the average distance from the table, each block of 2^{17} routes corresponds to 19,660,800 units of accumulated distance.

All route quantities are even.

The numbers in the route column are all even: 2, 2, 2, 6, 16, 24,...

One possible explanation is the symmetry between a route and its reverse route. If a route has distance D, the route traveled in the opposite direction also has the same distance D. Therefore, the routes appear in pairs.

This also explains why the total is even: 362,880 = 2 × 181,440

CONCLUSION

It is concluded that the possibility of facilitating pre-analysis is demonstrated by the explanation and ratification of the article in the bibliographic references [1], in order to better address the outcome of the same article to facilitate a betterComprehension and understanding of the Mathematical Theory raised and proposed in 2023. With the aim of facilitating the analysis of NP-hard problems such as the Traveling Salesman Problem. The question is...The types of questions that can be raised and explored in future research in order to better simplify the study and the conclusions of the outcomes.From mathematical theories. As the problem in question is considered NP-hard, the article proposes an alternative approach to the problem in a way that simplifies arduous and complex questions in mathematics. Remember that the most important results of the articles were presented, and how could the questions be analyzed? Simply, for example, by using the properties of prime numbers in a generalized way, comparing them with the output of the table.of the program resultsresults.txt, the properties of the numbersGeneralized primes, whether due to patterns in prime numbers, or as properties of Physics or Mathematics such as Twin Primes, are a relevant consideration that was addressed in the route of the 10th city which measures in hundreds andthousandsthedistance between the cities and the total route of the proposed problem.

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